

Inequalities Review

1. a) $x = 10$

$$\begin{aligned} 3x - 2 &< 13 \\ 3(10) - 2 &< 13 \\ 30 - 2 &< 13 \\ 28 &< 13 \end{aligned}$$

False

$x = 10$ does NOT satisfy the inequality

b) $x = -2$

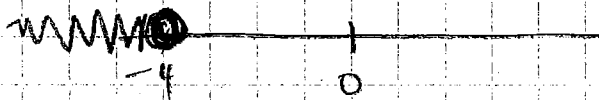
$$\begin{aligned} 2(3 - x) - 1 &\geq 7 \\ 2(3 - (-2)) - 1 &\geq 7 \\ 2(3 + 2) - 1 &\geq 7 \\ 2(5) - 1 &\geq 7 \\ 10 - 1 &\geq 7 \\ 9 &\geq 7 \end{aligned}$$

True

$x = -2$ Does satisfy the inequality

2. a) $-3x + 2 \geq x + 18$
 $-4x + 2 \geq 18$
 $-4x \geq 16$

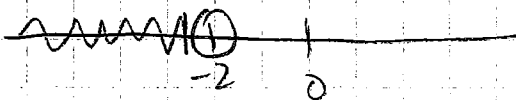
$$x \leq -4$$



$(-\infty, -4]$ interval

$\{x \mid x \leq -4, x \in \mathbb{R}\}$ set

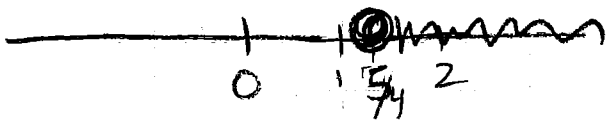
b) $2(x + 3) < x + 4$
 $2x + 6 < x + 4$
 $x + 6 < 4$
 $x < -2$



$(-\infty, -2)$ interval

$\{x \mid x < -2, x \in \mathbb{R}\}$ set

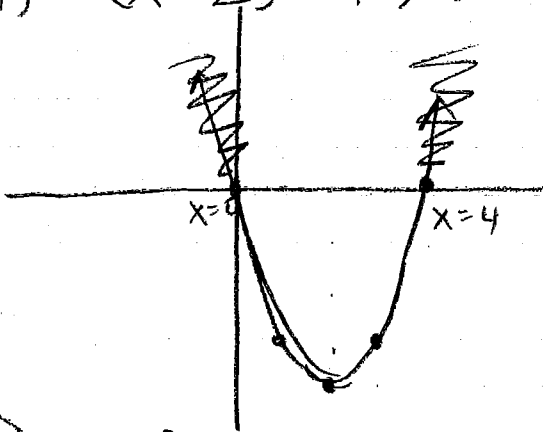
$$\begin{aligned}
 c) \quad & 3(2x-1) \geq 2(1+x) \\
 & 6x-3 \geq 2+2x \\
 & 4x-3 \geq 2 \\
 & 4x \geq 5 \\
 & x \geq 5/4
 \end{aligned}$$



$$\left[\frac{5}{4}, \infty \right) \\
 \{x \mid x \geq \frac{5}{4} \quad x \in \mathbb{R}\}$$

$$3. a) (x-2)^2 - 4 > 0$$

greater (above x-axis)



$$(-\infty, 0) \cup (4, \infty)$$

$$\{x \mid x < 0 \text{ or } x > 4 \quad x \in \mathbb{R}\}$$

$$b) \quad 3x^2 + 12x + 9 < 0$$

$$3(x^2 + 4x + 4) + 9 - 3(4) < 0$$

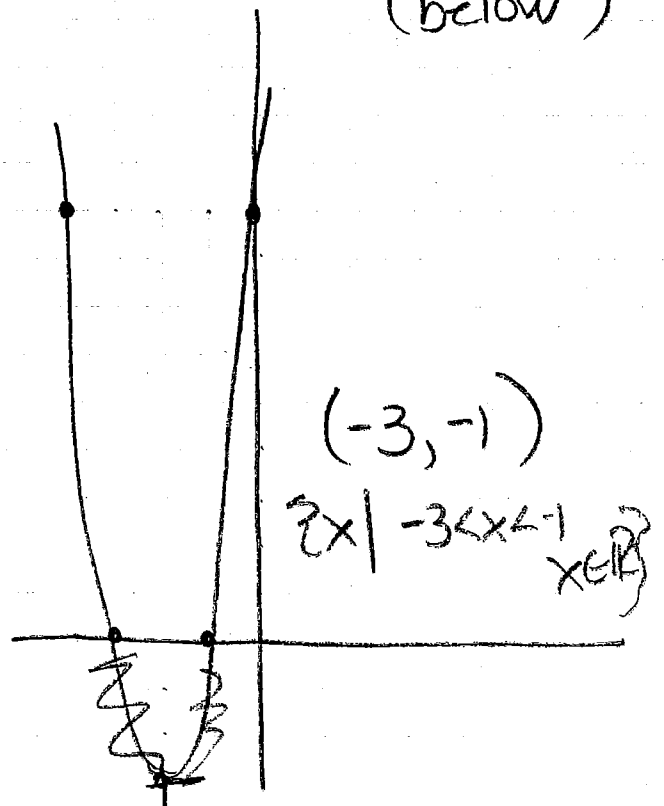
less (below)

$$\left(\frac{4}{2}\right)^2 = 2^2 = 4$$

$$\begin{aligned}
 3(x+2)^2 + 9 - 12 < 0 \\
 3(x+2)^2 - 3 < 0
 \end{aligned}$$

a=3
mult by 3

0	0	0
1	x	3
2	4	12



$$(-3, -1)$$

$$\{x \mid -3 < x < -1 \quad x \in \mathbb{R}\}$$

c) $-x^2 + 2x \geq 0$ greater or (above or) on ≥ 0
 $-(x^2 - 2x + 1) - (-1) \geq 0$

$$\left(-\frac{2}{2}\right)^2 - (-1) = 1$$

$$-(x-1)^2 + 1 \geq 0$$



$$[0, 2]$$

$$\{x \mid 0 \leq x \leq 2, x \in \mathbb{R}\}$$

4a) $x^2 - 2x > 63$
 $x^2 - 2x - 63 > 0$
 $(x-9)(x+7) > 0$

$$\frac{-x}{-9} = -63$$

$$\frac{-x}{7} = -2$$

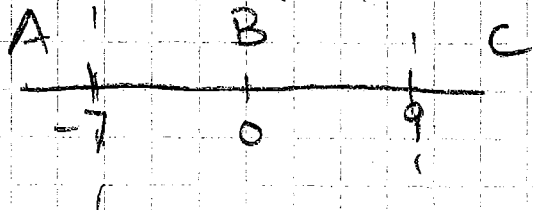
Roots

$$x-9=0$$

$$x+7=0$$

$$x=9$$

$$x=-7$$



A use $x=-8$
 $(-8)^2 - 2(-8) > 63$
 $64 + 16 > 63$
 $80 > 63$
 T

B use $x=0$
 $0^2 - 2(0) > 63$
 $0 > 63$
 F

C use $x=10$
 $10^2 - 2(10) > 63$
 $100 - 20 > 63$
 $80 > 63$
 T

$$(-\infty, -7) \cup (9, \infty)$$

$$\{x \mid x < -7 \text{ or } x > 9, x \in \mathbb{R}\}$$

$$4b \quad 2x^2 - 7x - 30 \geq 0$$

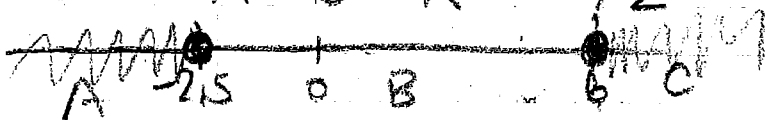
$$2x^2 - 12x + 5x - 30 \geq 0$$

$$2x(x-6) + 5(x-6) \geq 0$$

$$(x-6)(2x+5) \geq 0$$

$$\frac{-x}{-12} = \frac{-60}{5} = -7$$

roots $x = -6$ $x = -5/2$



A use $x = -3$

$$2(-3)^2 - 7(-3) - 30 \geq 0$$

$$18 + 21 - 30 \geq 0$$

$$9 \geq 0$$

T

B use $x = 0$

$$2(0)^2 - 7(0) - 30 \geq 0$$

$$-30 \geq 0$$

F

C use $x = 7$

$$2(7)^2 - 7(7) - 30 \geq 0$$

$$98 - 49 - 30 \geq 0$$

$$19 \geq 0$$

T

$$(-\infty, -5/2] \cup [6, \infty)$$

$$\{x \mid x \leq -5/2 \text{ or } x \geq 6, x \in \mathbb{R}\}$$

5a $x^2 + 8x - 48 < 0$

$$(x+12)(x-4) < 0$$

roots $x = -12$ $x = 4$

	use $x = -100$	$x = 0$	$x = 100$
$(x+12)$	\ominus	\oplus	\oplus
$x-4$	\ominus	\ominus	\oplus
$(x+12)(x-4)$	\oplus	\ominus	\oplus

$$(-12, 4)$$

$$\{x \mid -12 < x < 4, x \in \mathbb{R}\}$$

When is the parabola less than zero (below x-axis) (Negative.)

$$5b \quad x(6x+5) \geq 4$$

$$6x^2 + 5x \geq 4$$

$$6x^2 + 5x - 4 \geq 0$$

$$\frac{-x \pm \sqrt{24}}{6} = -\frac{24}{6}$$

$$6x^2 + 8x - 3x - 4 \geq 0$$

$$2x(3x+4) - 1(3x+4) \geq 0$$

$$(3x+4)(2x-1) \geq 0$$

roots $x = -\frac{4}{3}$ $x = \frac{1}{2}$

	use $x = -100$	use $x = 0$	use $x = 100$
$(3x+4)$	\ominus	\oplus	\oplus
$(2x-1)$	\ominus	\ominus	\oplus
$(3x+4)(2x-1)$	\oplus	\ominus	\oplus

When is the parabola above or on x-axis
(positive)

$$(-\infty, -\frac{4}{3}] \cup [\frac{1}{2}, \infty)$$

$$\{x \mid x \leq -\frac{4}{3} \text{ or } x \geq \frac{1}{2}\}$$