Integration By Parts Review

Thursday, April 6, 2017 8:36 AM

AP Calculus
8.1 Review

$\int u\,dv = uv - \int v\,du$

Integration By Parts Review

Evaluate the integral	Evaluate the integral
$\int xe^{-2x}dx$ u = x du = dx dv = $e^{-2x}dx$ v = $-\frac{1}{2}e^{-2x}$	$= x\cdot(-\frac{1}{2}e^{-2x}) - \int -\frac{1}{2}e^{-2x}\cdot dx$ $= \frac{-xe^{-2x}}{2} + \frac{1}{2}\int e^{-2x}dx$ $= \frac{-xe^{-2x}}{2} + \frac{1}{2}\cdot(-\frac{1}{2})e^{-2x} + C$ $= -\frac{xe^{-2x}}{2} - \frac{e^{-2x}}{4} + C$ $= \frac{-e^{-2x}(2x+1)}{4} + C$
$\int x^2e^{x^3}dx$ $u = x^3$ $\frac{du}{dx} = 3x^2$ $\frac{du}{3} = x^2dx$	$\int e^u\cdot\frac{du}{3}$ $= \frac{1}{3}\cdot e^u + C$ $= \frac{1}{3}e^{x^3} + C$
$\int t\ln(t+1)dt$ $u = \ln(t+1)$ $du = \frac{1}{t+1}dt$ $dv = t\,dt$ $v = \frac{t^2}{2}$	$= \ln(t+1)\cdot\frac{t^2}{2} - \int\frac{1}{2}t^2\cdot\frac{1}{t+1}dt$ $= \frac{t^2\ln(t+1)}{2} - \frac{1}{2}\int\frac{t^2-1+1}{t+1}dt$ $= \frac{t^2\ln(t+1)}{2} - \frac{1}{2}\int\frac{t^2-1}{t+1} + \frac{1}{t+1}dt$ $= \frac{t^2\ln(t+1)}{2} - \frac{1}{2}\int\frac{(t+1)(t-1)}{t+1} + \frac{1}{t+1}dt$ $= \frac{t^2\ln(t+1)}{2} - \frac{1}{2}\int t-1+\frac{1}{t+1}dt$ $= \frac{t^2\ln(t+1)}{2} - \frac{1}{2}\left[\frac{t^2}{2} - t + \ln\lvert t+1\rvert\right] + C$

$= \frac{t^2 \ln(t+1)}{2} - \frac{t^2}{4} + \frac{t}{2} - \frac{1}{2}\ln|t+1| + C$

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$\int \frac{(\ln x)^2}{x} dx$

$u = \ln x$
$du = \frac{1}{x} dx$

$= \int u^2 du$
$= \frac{1}{3}u^3 + C$
$= \frac{(\ln x)^3}{3} + C$

$\int (x^2 - 1)e^x dx$

$u = x^2 - 1$
$du = 2x\,dx$
$dv = e^x dx$
$v = e^x$

$u = 2x$
$du = 2dx$
$dv = e^x dx$
$v = e^x$

$= (x^2-1)e^x - \int e^x \cdot 2x\,dx$
$= e^x(x^2-1) - \left[2xe^x - \int e^x \cdot 2dx\right]$
$= e^x(x^2-1) - 2xe^x + 2e^x + C$
$= e^x\left[x^2 - 1 - 2x + 2\right] + C$
$= e^x\left[x^2 - 2x + 1\right] + C$
$= e^x(x-1)^2 + C$

$\int x\sqrt{x-1}\,dx$

$u = x$
$du = dx$
$dv = (x-1)^{1/2} dx$
$v = \frac{2}{3}(x-1)^{3/2}$

$= x \cdot \frac{2}{3}(x-1)^{3/2} - \int \frac{2}{3}(x-1)^{3/2} \cdot dx$
$= \frac{2x(x-1)^{3/2}}{3} - \frac{2}{3} \cdot \frac{2}{5}(x-1)^{5/2} + C$
$= \frac{10x(x-1)^{3/2}}{15} - \frac{4(x-1)^{5/2}}{15} + C$
$= \frac{2(x-1)^{3/2}}{15} \cdot \left[5x - 2(x-1)\right] + C$
$= \frac{2(x-1)^{3/2}(3x+2)}{15} + C$