

7.1 Linear Inequalities in One Variable

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7.1 Linear Inequalities in One Variable

An inequality tells us the relationship between two numbers or expressions.

A. Operations with Inequalities

When you multiply or divide by a negative number the inequality changes direction

Original Inequality	Operation	Resulting Inequality	
$9 > 6$	Add 3	$9 + 3 \blacksquare 6 + 3$	$12 > 9$
$9 > 6$	Subtract 3	$9 - 3 \blacksquare 6 - 3$	$6 > 3$
$9 > 6$	Multiply by 3	$9 \times 3 \blacksquare 6 \times 3$	$27 > 18$
$9 > 6$	Multiply by -3	$9 \times -3 \blacksquare 6 \times -3$	$-27 < -18$
$9 > 6$	Divide by 3	$\frac{9}{3} \blacksquare \frac{6}{3}$	$3 > 2$
$9 > 6$	Divide by -3	$\frac{9}{-3} \blacksquare \frac{6}{-3}$	$-3 < -2$

Similar results are observed for inequalities that include the symbols $<$, \leq and \geq .

B. Solving an Inequality

To solve an inequality means to find the values of the variable that make the inequality true.

Example 1: Solve each inequality and graph its solution. Express your answer in set notation.

a) $3x - 4 \geq 5$

$$\begin{aligned} &+4 \quad +4 \\ 3x &\geq 9 \\ \frac{3x}{3} &\geq \frac{9}{3} \\ x &\geq 3 \end{aligned}$$



$$\{x \mid x \geq 3 \quad x \in \mathbb{R}\}$$

b) $4x - 1 > x + 5$

$$\begin{aligned} 4x &> x + 6 \\ 3x &> 6 \\ x &> 2 \end{aligned}$$



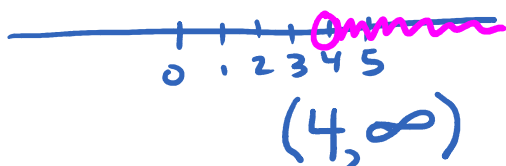
$$\{x \mid x > 2 \quad x \in \mathbb{R}\}$$

Example 2: Solve the following inequalities algebraically. Express your final answer in interval notation.

a) $6x - 3(x+1) > x + 5$

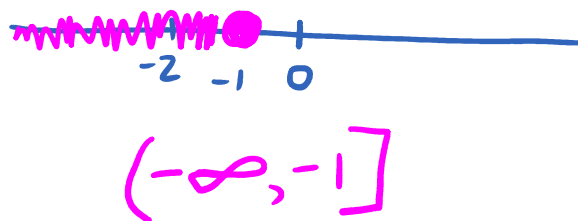
$$\begin{aligned} 6x - 3x - 3 &> x + 5 \\ 3x - 3 &> x + 5 \\ 2x - 3 &> 5 \\ 2x &> 8 \\ x &> 4 \end{aligned}$$

$\{x \mid x > 4, x \in \mathbb{R}\}$



b) $2(3 - x) - 1 \geq 7$

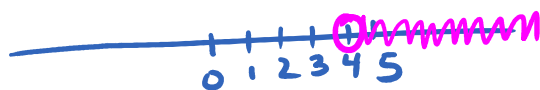
$$\begin{aligned} 6 - 2x - 1 &\geq 7 \\ -2x + 5 &\geq 7 \\ -2x &\geq 2 \\ x &\leq -1 \end{aligned}$$



Example 3: Solve the following inequalities algebraically. Answer in both set notation and interval notation.

a) $\frac{3x}{4} + \frac{x}{2} > 5$

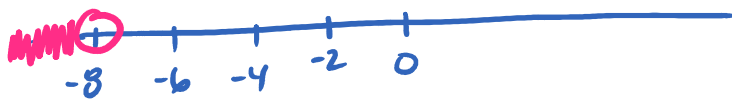
$$\begin{aligned} 4\left(\frac{3x}{4}\right) + 4\left(\frac{x}{2}\right) &> 4(5) \\ 3x + 2x &> 20 \\ 5x &> 20 \\ x &> 4 \end{aligned}$$



set $\{x \mid x > 4, x \in \mathbb{R}\}$
interval $(4, \infty)$

b) $\frac{3+x}{2} < \frac{x-2}{4}$

$$\begin{aligned} 4\left(\frac{3+x}{2}\right) &< 4\left(\frac{x-2}{4}\right) \\ 6 + 2x &< x - 2 \\ 6 + x &< -2 \\ x &< -8 \end{aligned}$$



$\{x \mid x < -8, x \in \mathbb{R}\}$
 $(-\infty, -8)$