Exponent and Logarithm Review

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Exponents and Logarithms

Exponent Laws:

$x^m \cdot x^n = x^{m+n}$

$\frac{x^m}{x^n} = x^{m-n}$

$(x^m)^n = x^{mn}$

$x^0 = 1$

$x^{-n} = \frac{1}{x^n}$

$x^{\frac{m}{n}} = \sqrt[n]{x^m}$

Solving Exponential Equations

1. $8^{2x-3} = 16^{x-1}$

$(2^3)^{2x-3} = (2^4)^{x-1}$

$2^{6x-9} = 2^{4x-4}$

$6x - 9 = 4x - 4$

$2x = 5$

$x = 5/2$

Write in the same base

Make exponents =

Solve

2. $2^{x^2} = (16^{x-1}) \times 2^x$

$2^{x^2} = (2^4)^{x-1} \cdot 2^x$

$2^{x^2} = 2^{4x-4} \cdot 2^x$

$2^{x^2} = 2^{5x-4}$

$x^2 = 5x - 4$

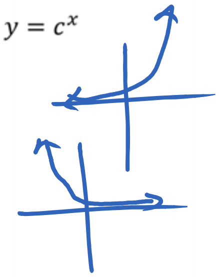
$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x = 4 \quad x = 1$

AP Calculus

<u>Graphing Exponential Equations</u> $y = c^x$



When $c > 1$ increasing

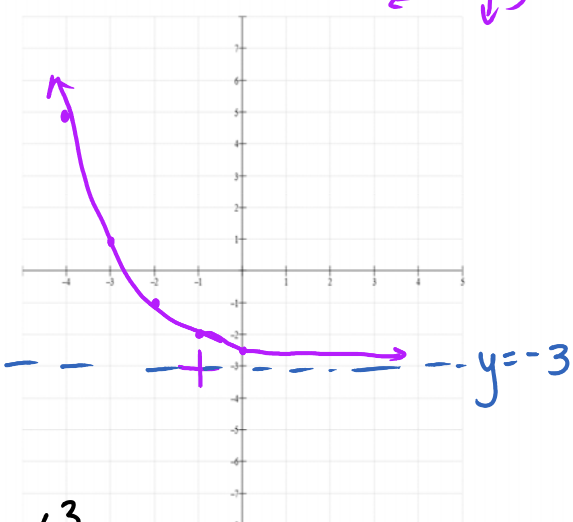
$0 < c < 1$ decreasing

$c > 0$

$y = \left(\frac{1}{2}\right)^x$

x	y
1	$\frac{1}{2}$
0	1
-1	2
-2	4
-3	8

3. Graph $y = \left(\frac{1}{2}\right)^{x+1} - 3$ ← ↓3



Domain $\{x : x \in \mathbb{R}\}$

Range $\{y : y > -3 \ y \in \mathbb{R}\}$

Asymptote $y = -3$

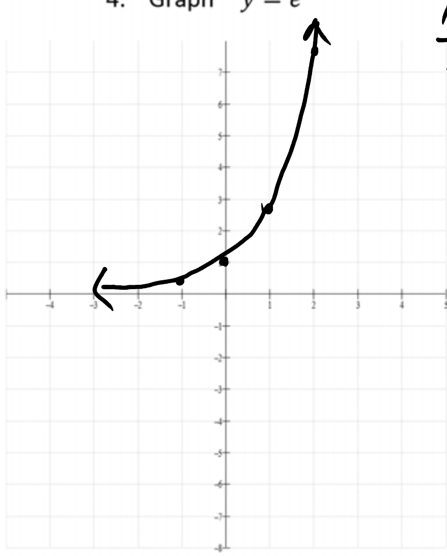
Intercepts

y-int
$x = 0$
$y = \left(\frac{1}{2}\right)^{0+1} - 3$
$y = \frac{1}{2} - 3$
$y = -\frac{5}{2}$
$y = -2.5$

x-int $y = 0$
$0 = \left(\frac{1}{2}\right)^{x+1} - 3$
$3 = \left(\frac{1}{2}\right)^{x+1}$
$\log 3 = \log\left(\frac{1}{2}\right)^{x+1}$
$\log 3 = (x+1)\log\frac{1}{2}$
$\frac{\log 3}{\log \frac{1}{2}} - 1 = x$
$x = -2.58$

$2 < e < 3$
$e \approx 2.7$

4. Graph $y = e^x$



x	y
-1	$e^{-1} = .4$
0	1
1	2.7
2	7.4

Domain $\{x : x \in \mathbb{R}\}$

Range $\{y : y > 0 \ y \in \mathbb{R}\}$

Asymptotes $y = 0$

Intercepts

x-int
None

y-int
$y = 1$

AP Calculus

Laws of Logarithms

$\log_c 1 = 0$

$\log_c c = 1$

$\log_c c^x = x$

$c^{\log_c x} = x$

$\log_c x + \log_c y = \log_c (x \cdot y)$

$\log_c x - \log_c y = \log_c \left(\frac{x}{y}\right)$

$\log_c x^p = p \log_c x$

$\log_c x = \frac{\log_m x}{\log_m c}$

$\log_e x = \ln x$

Inverses

$y = 2^x$

$\boxed{x = 2^y \quad y = \log_2 x}$

5. Evaluate

a) $\log_2 160 - \log_2 10$

$\log_2 \left(\frac{160}{10}\right)$

$\log_2 16$

$\log_2 2^4$

4

b) $\log_8 64^5$

$= 5 \log_8 64$

$= 5 \log_8 8^2$

$= 5(2)$

$= 10$

c) $\log_4 \frac{1}{64} + \log_4 1$

$\log_4 64^{-1} + 0$

$-1 \log_4 64$

$-1 \log_4 4^3$

$-1(3)$

$= -3$

6. Simplify $\log_2 x + \log_4 y$

$\log_2 x + \frac{\log_2 y}{\log_2 4}$

$\log_2 x + \frac{\log_2 y}{2}$

$\log_2 x + \frac{1}{2} \log_2 y$

$\log_2 x + \log_2 y^{1/2}$

$\log_2 (x y^{1/2})$ or $\log_2 (x\sqrt{y})$

7. Solve

a) $2^{x+2} = 5^{x-1}$

$$\log 2^{x+2} = \log 5^{x-1}$$
$$(x+2)\log 2 = (x-1)\log 5$$
$$x\log 2 + 2\log 2 = x\log 5 - \log 5$$
$$2\log 2 + \log 5 = x\log 5 - x\log 2$$
$$2\log 2 + \log 5 = x(\log 5 - \log 2)$$
$$\frac{2\log 2 + \log 5}{\log 5 - \log 2} = x$$
$$x = 3.27$$

b) $3 = 4e^{x+1}$

$$\frac{3}{4} = e^{x+1}$$

$\ln e = \log_e e$

$$\ln\left(\frac{3}{4}\right) = \ln e^{x+1}$$
$$\ln\left(\frac{3}{4}\right) = (x+1)\ln e$$
$$\ln\left(\frac{3}{4}\right) = (x+1)(1)$$
$$\ln\frac{3}{4} - 1 = x$$
$$x = -1.29$$

c) $\log_6(x-1) = 1 - \log_6(x-6)$

$$\log_6(x-1) + \log_6(x-6) = 1$$
$$\log_6((x-1)(x-6)) = 1$$
$$\log_6(x^2 - 7x + 6) = \log_6 6$$
$$x^2 - 7x + 6 = 6$$
$$x^2 - 7x = 0$$
$$x(x-7) = 0$$
$$\cancel{x = 0} \qquad \boxed{x = 7}$$

$x - 1 > 0$

$x = 0$

$0 - 1 > 0$

d) $5 = 2\log_3(2x+1)$

$$\frac{5}{2} = \log_3(2x+1)$$
$$3^{\frac{5}{2}} = 2x + 1$$
$$3^{5/2} - 1 = 2x$$
$$\frac{3^{5/2} - 1}{2} = x$$
$$x = 7.29$$