## 5.1 Graphing Sine and Cosine Functions

*The Graph of* $f(\theta) = \sin\theta$ $\sin\theta = \frac{y}{r}$

unit circle

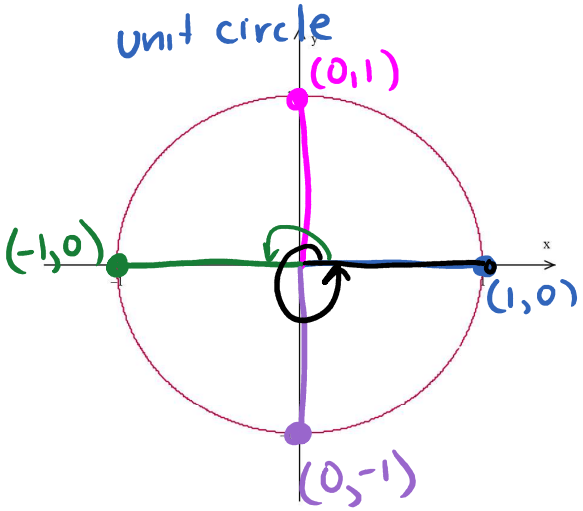
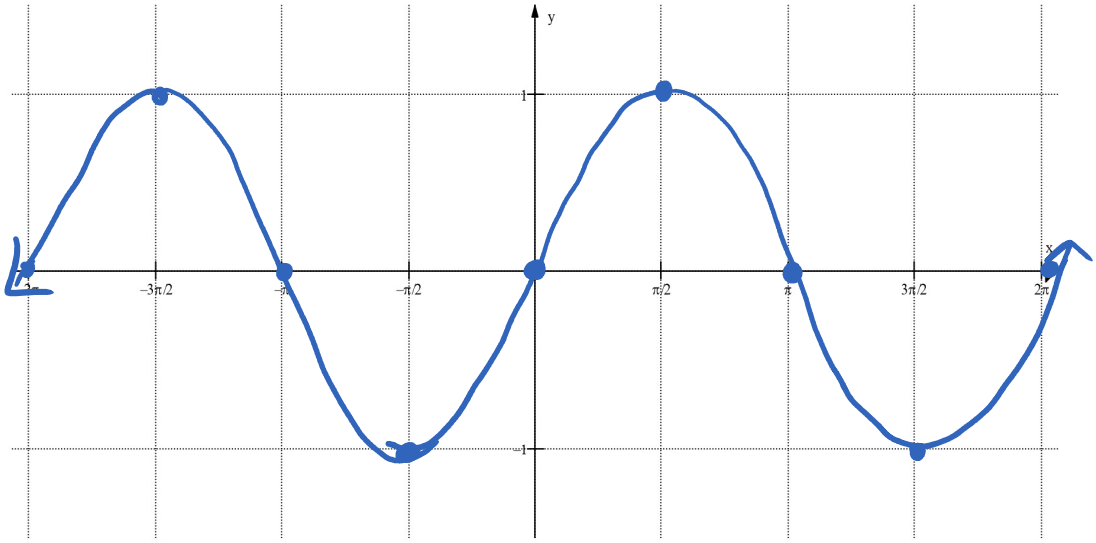



Table of Values

| $\theta$ | $f(\theta)$ |
|---|---|
| $0$ | $\frac{0}{1} = 0$ |
| $\pi/2$ | $\frac{1}{1} = 1$ |
| $\pi$ | $\frac{0}{1} = 0$ |
| $3\pi/2$ | $\frac{-1}{1} = -1$ |
| $2\pi$ | $\frac{0}{1} = 0$ |



Pre-Calculus 12

*The Graph of* ~~$y = \cos\theta$~~ $f(\theta) = \cos\theta$ $\cos\theta = \frac{x}{r}$

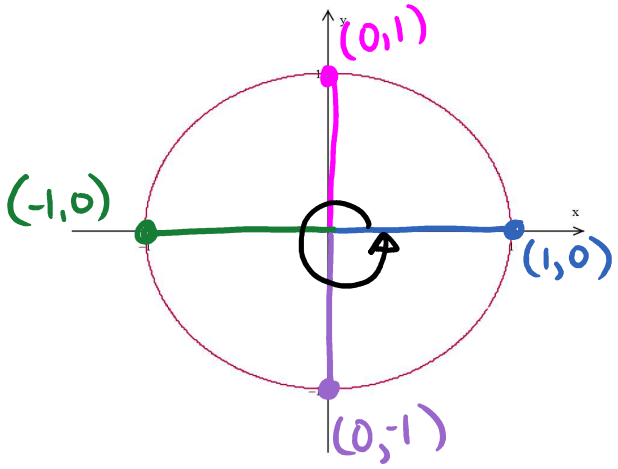
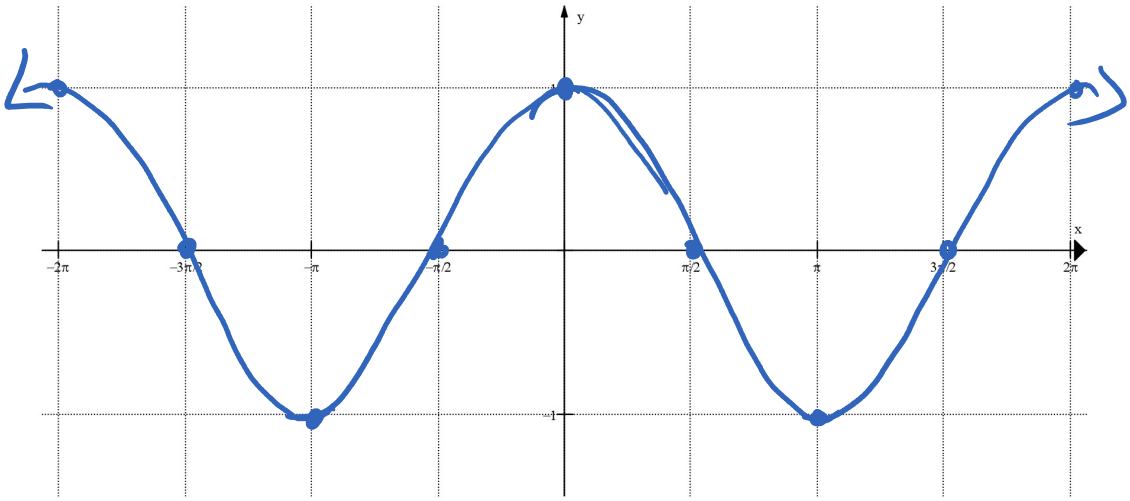



Table of Values

| $\theta$ | $f(\theta)$ |
|---|---|
| 0 | $y_1 = 1$ |
| $\pi/2$ | $0/1 = 0$ |
| $\pi$ | $-y_1 = -1$ |
| $3\pi/2$ | $0/1 = 0$ |
| $2\pi$ | $y_1 = 1$ |




Functions that repeat themselves over a particular interval are called periodic. The interval is called the period. The amplitude of a periodic function is one half the difference between the maximum and minimum values.

$$Amplitude = \frac{|\max - \min|}{2}$$

**Ex. #1:** What is the amplitude and period of $y = \sin\theta$ and $y = \cos\theta$ ?

$y = \sin\theta$

$Amp = \frac{|1-(-1)|}{2}$ period $= 2\pi$

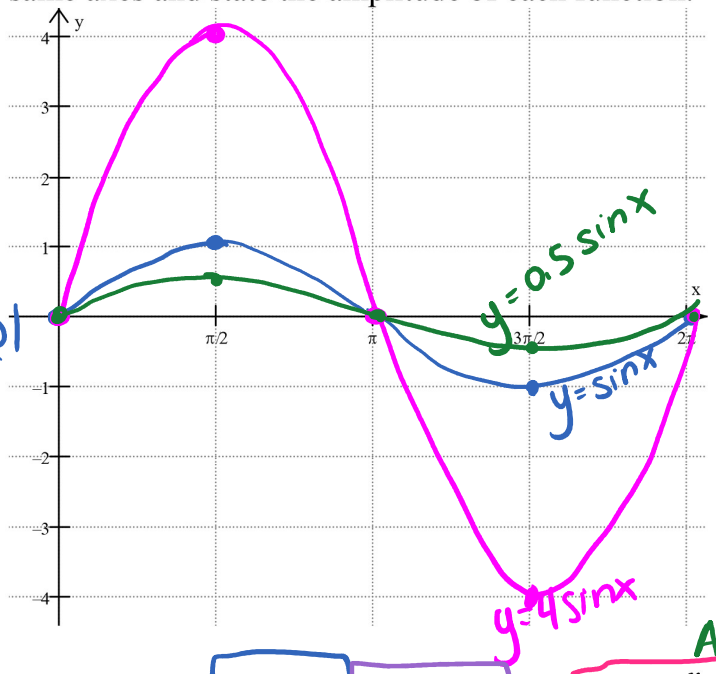
$Amp = \frac{2}{2} = 1$

$y = \cos\theta$

$Amp = \frac{|1-(-1)|}{2}$ period $= 2\pi$

$Amp = \frac{2}{2} = 1$

**Ex. #2:** Graph $y = \sin x$, $y = 4\sin x$, and $y = 0.5\sin x$ for $0 \le x \le 2\pi$ on the same axes and state the amplitude of each function.




$y = \sin x$

| | |
|---|---|
| 0 | 0 |
| $\pi/2$ | 1 |
| $\pi$ | 0 |
| $3\pi/2$ | −1 |
| $2\pi$ | 0 |

$\text{Amp} = \left|\frac{1-(-1)}{2}\right|$

$\text{Amp} = 1$

$y = 4\sin x$

$a = 4$

Mult y's by (4)

| | |
|---|---|
| 0 | 0 |
| $\pi/2$ | 4 |
| $\pi$ | 0 |
| $3\pi/2$ | −4 |
| $2\pi$ | 0 |

$\text{Amp} = \left|\frac{4-(-4)}{2}\right| = \frac{8}{2} = 4$

$y = 0.5\sin x$

$a = 0.5$

Mult y's by (0.5)

| | |
|---|---|
| 0 | 0 |
| $\pi/2$ | 0.5 |
| $\pi$ | 0 |
| $3\pi/2$ | −0.5 |
| $2\pi$ | 0 |

$\text{Amp} = \left|\frac{0.5-(-0.5)}{2}\right| = 0.5$

**Ex. #3:** Graph $y = \cos x$, $y = \cos 2x$, and $y = \cos\frac{x}{2}$ for $0 \le x \le 2\pi$ on the same axes and state the period of each function.


$y = \cos x$

| | |
|---|---|
| 0 | 1 |
| $\pi/2$ | 0 |
| $\pi$ | −1 |
| $3\pi/2$ | 0 |
| $2\pi$ | 1 |

period = $2\pi$

$y = \cos 2x$

$b = 2$

divide x's by (2)

period = $\pi$

| | |
|---|---|
| $0 = 0/2$ | 1 |
| $\frac{\pi}{4} = \frac{\pi}{2} \div 2$ | 0 |
| $\frac{2\pi}{4} = \frac{\pi}{2}$ | −1 |
| $\frac{3\pi}{4} = \frac{3\pi}{2} \div 2$ | 0 |
| $\frac{4\pi}{4} = \pi = \frac{2\pi}{2}$ | 1 |

$y = \cos\frac{x}{2}$

$y = \cos\frac{1}{2}x$

$b = \frac{1}{2}$

divide x-values by $(\frac{1}{2})$

| | |
|---|---|
| 0 | 1 |
| $\pi$ | 0 |
| $2\pi$ | −1 |
| $3\pi$ | 0 |
| $4\pi$ | 1 |

period = $4\pi$

**Note:** For functions of the form $y = a\sin b\theta$ and $y = a\cos b\theta$, where $a, b \neq 0$, the amplitude is $|a|$ and the period is $\frac{2\pi}{b}$ or $\frac{360^\circ}{b}$

**Ex. #4:** State the amplitude and period of the following functions in radians:

(a) $y = 2\cos\frac{x}{3}$ $b = \frac{1}{3}$

$\text{Amp} = |2| = 2$

$\text{period} = \frac{2\pi}{b} = \frac{2\pi}{1/3} = 2\pi \cdot \frac{3}{1} = 6\pi$

(b) $y = -4\sin 4x$ $b = 4$

$\text{Amp} = |-4| = 4$

$\text{period} = \frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$

**Ex. #5:** State the amplitude and period of the following functions in degrees:

(a) $y = \frac{1}{2}\sin 2\theta$ $b = 2$

$\text{Amp} = |\frac{1}{2}| = \frac{1}{2}$

$\text{period} = \frac{360^\circ}{2}$

$\text{period} = 180^\circ$

(b) $y = -\cos\frac{2\theta}{3}$ $b = 2/3$

$\text{Amp} = |-1| = 1$

$\text{period} = \frac{360^\circ}{2/3} = 360^\circ \cdot \frac{3}{2} = 540^\circ$

**Ex. #6:** Write an equation of the given function with the following characteristics:

(a) sine function

amplitude = 3 $a = 3$

period = $\frac{\pi}{4}$

$\text{period} = \frac{2\pi}{b}$

$4b \cdot \frac{\pi}{4} = \frac{2\pi}{b} \cdot 4b$

$b\pi = 8\pi$

$b = 8$

$y = 3\sin 8\theta$

(b) cosine function

amplitude = $\frac{1}{3}$ $a = \frac{1}{3}$

period = 60°

$\text{period} = \frac{360^\circ}{b}$

$b\,60^\circ = \frac{360^\circ}{b} \cdot b$

$b60^\circ = 360^\circ$

$b = 6$

$y = \frac{1}{3}\cos 6\theta$

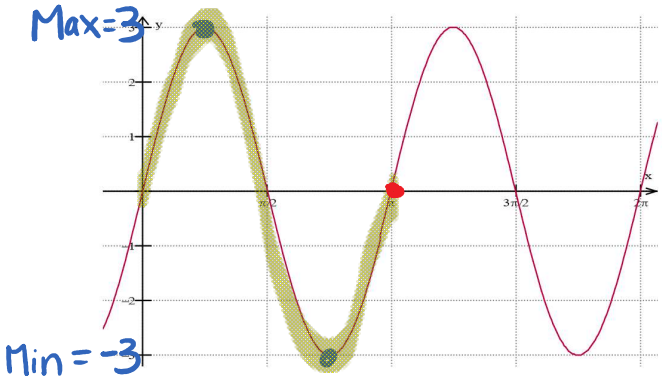
$b = 8$

$b60 - 360$

$b = 6$

$y = \frac{1}{3}\cos 60$

**Ex. #7:** Determine the equation for the following sine function.




$Amp = \frac{|3-(-3)|}{2} = \frac{6}{2} = 3$

period = $\pi$

period = $\frac{2\pi}{b}$

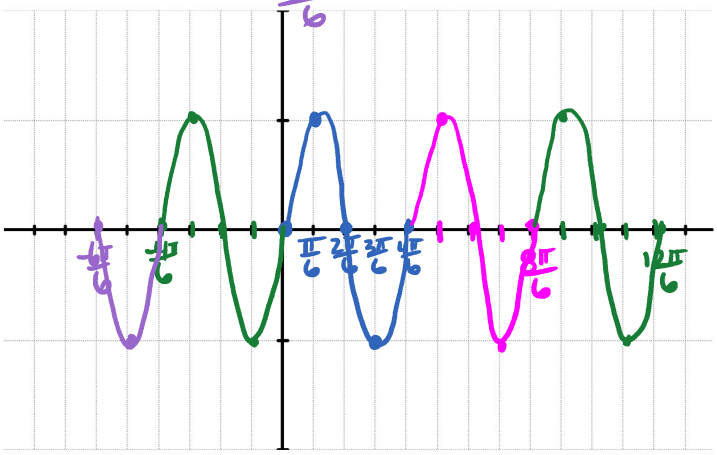
$\pi = \frac{2\pi}{b}$

$b = 2$

$y = 3\sin 2\theta$

**Ex. #8:** Sketch the graph of the following functions.

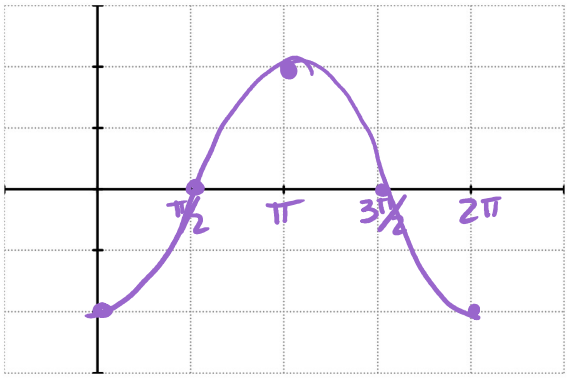
(a) $y = \sin 3x, \quad -\pi \le x < 2\pi$ $= \frac{12\pi}{6}$ $b=3$ $-\frac{6\pi}{6}$



b = 3 divide x's by 3

| | |
|---|---|
| $0 = 0/3$ | 0 |
| $\frac{\pi}{6} = \pi/2 \div 3$ | 1 |
| $\frac{2\pi}{6} = \pi/3$ | 0 |
| $\frac{3\pi}{6} = \frac{3\pi}{2} \div 3$ | -1 |
| $\frac{4\pi}{6} = \frac{2\pi}{3}$ | 0 |

(b) $y = -2\cos x, \quad 0 \le x \le 2\pi$




a = -2

Mult y's by (-2)

| | |
|---|---|
| 0 | -2 |
| $\pi/2$ | 0 |
| $\pi$ | 2 |
| $3\pi/2$ | 0 |
| $2\pi$ | -2 |