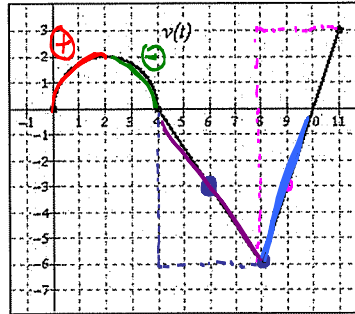


Worksheet 5. Sample Practice Problems for the Topic of Motion (Part 2)

Example 1 (graphical).

The graph to the right shows the velocity,  $v(t)$ , of a particle moving along the  $x$ -axis for  $0 \leq t \leq 11$ . It consists of a semicircle and two line segments. Use the graph and your knowledge of motion to answer the following questions.



1. At what time  $t$ ,  $0 \leq t \leq 11$ , is the speed of the particle the greatest?

speed = |velocity|  $t = 8$

2. At which of the times,  $t = 2$ ,  $t = 6$  or  $t = 9$ , is the acceleration of the particle the greatest? Explain your answer.

$a(t) = v'(t)$   
 slope of  $v(t)$   
 $v'(2) = 0$      $v'(6) = -\frac{6}{4} = -\frac{3}{2}$      $v'(9) = \frac{9}{3} = 3$   
 $t = 9$

3. Over what time intervals is the particle moving to the left? Explain your answer.

left if  $v(t) < 0$   
 $4 < t < 10$

4. Over what time intervals is the speed of the particle decreasing? Explain your answer.

speed decreasing if  $a(t)$  and  $v(t)$  have opposite signs  
 $2 < t < 4$      $8 < t < 10$

5. Find the total distance traveled by the particle over the time interval  $0 \leq t \leq 11$ .

$$\int_0^{11} |v(t)| dt = \frac{\pi(2)^2}{2} + \frac{(6)(6)}{2} + \frac{(1)(3)}{2} = 2\pi + 18 + \frac{3}{2}$$

$$= 2\pi + \frac{36}{2} + \frac{3}{2} = 2\pi + \frac{39}{2}$$

6. Find the value of  $\int_0^{11} v(t) dt$  and explain the meaning of this integral in the context of the problem.

change in position  $\int_0^{11} v(t) dt = \frac{\pi(2)^2}{2} + \frac{(6)(-6)}{2} + \frac{(1)(3)}{2}$

$$= 2\pi - \frac{36}{2} + \frac{3}{2} = 2\pi - \frac{33}{2}$$

$$\int_0^{11} v(t) dt = x(11) - x(0)$$

7. If the initial position of the particle is  $x(0) = 2$ , find the position of the particle at time  $t = 11$ .

$$x(11) = \int_0^{11} v(t) dt + x(0) \quad x(11) = 2\pi - \frac{29}{2}$$

$$= 2\pi - 3\frac{3}{2} + 2$$

**Example 2 (analytic/graphical/calculator active).**

The rate of change, in kilometers per hour, of the altitude of a hot air balloon is given by  $r(t) = t^3 - 4t^2 + 6$  for time  $0 \leq t \leq 4$ , where  $t$  is measured in hours. Assume the balloon is initially at ground level.

1. For what values of  $t$ ,  $0 \leq t \leq 4$ , is the altitude of the balloon decreasing?

$$r(t) = 0 \quad (0, 1.57) \quad (1.57, 3.54) \quad (3.54, 4)$$

$$t = 1.57 \quad t = 3.54 \quad \oplus \quad \ominus \quad \oplus$$

$$1.57 < t < 3.54$$

2. Find the value of  $r'(2)$  and explain the meaning of the answer in the context of the problem. Indicate units of measure.

$$r'(t) = 3t^2 - 8t$$

$$r'(2) = -4 \text{ km/h}^2$$

3. What is the altitude of the balloon when it is closest to the ground during the time interval  $2 \leq t \leq 4$ ?

altitude is min  $t = 3.54$   $x(3.54)$

$$\int_0^{3.54} r(t) dt = y(3.54) - y(0) \quad 1.348$$

$$y(3.54) = \int_0^{3.54} r(t) dt + 0 = 1.348$$

4. Find the value of  $\int_0^4 r(t) dt$  and explain the meaning of the answer in the context of the problem. Indicate units of measure.

$$\int_0^4 r(t) dt = 2.667$$

Change in altitude over  $0 \leq t \leq 4$

5. Find the value of  $\int_0^4 |r(t)| dt$  and explain the meaning of the answer in the context of the problem. Indicate units of measure.

$$11.529$$

total amount travelled in altitude

Max  $v(t) = 0$

$t = 1.57$   $t = 3.54$   
Min

Endpoints  $t = 0$   $t = 4$

Max at  $t = 1.57$

6. What is the maximum altitude of the balloon during the time interval  $0 \leq t \leq 4$ ?

$t = 0$   
 $y(0) = 0$

$t = 1.57$

$\int_0^{1.57} v(t) dt = y(1.57) - y(0)$   
 $y(1.57) = 5.779$

$\int_0^4 v(t) dt = y(4) - y(0)$   
 $y(4) = 2.667$

**Example 3 (numerical).**

The table below gives values for the velocity and acceleration of a particle moving along the  $x$ -axis for selected values of time  $t$ . Both velocity and acceleration are differentiable functions of time  $t$ . The velocity is decreasing for all values of  $t$ ,  $0 \leq t \leq 10$ . Use the data in the table to answer the questions that follow.

Time, $t$	0	2	6	10
Velocity, $v(t)$	5	3	-1	-8
Acceleration, $a(t)$	0	-1	-3	-5

1. Is there a time  $t$  when the particle is at rest? Explain your answer.
2. At what time indicated in the table is the speed of the particle decreasing? Explain your answer.
3. Use a left Riemann sum to approximate  $\int_0^{10} v(t) dt$ . Show the computations you use to arrive at your answer. Explain the meaning of the definite integral in the context of the problem.

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4. Is the approximation found in part (3) greater than or less than the actual value of the definite integral shown below? Explain your reasoning.

$$\int_0^{10} v(t) dt$$

5. Approximate the value of  $\int_0^{10} |v(t)| dt$  using a trapezoidal approximation with the three sub-intervals indicated by the values in the table. Show the computations you use to arrive at your answer. Explain the meaning of the definite integral in the context of the problem.

6. Determine the value of  $\int_0^{10} a(t) dt$ . Explain the meaning of the definite integral in the context of the problem.

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