5.2 New

Wednesday, March 16, 2022 2:58 PM

Pre-Calculus 12

5.2 Transformations of Sinusoidal Functions

The principles of transformations from Chapter 1 can be applied to the trigonometric functions.

Equation	What it was called	Sin/Cos equivalent	What it does...
$y = af(x)$	Vertical Stretch	y=asinx y=acosx	change of amplitude
$y = f(bx)$	Horizontal Stretch	y=sin(bx) y=cos(bx)	change of period
$y = f(x)+k$	Vertical Translation	y=sinx + k y=cosx + k	vertical displacement
$y = f(x+h)$	Horizontal Translation	y=sin(x + h) y=cos(x + h)	phase shift

Any sine or cosine function can be expressed in the form

$y = a\sin b(x-h) + k$ Or $y - k = a\cos(b(x-h))$

Amplitude = $|a|$

Period = $\frac{2\pi}{b}$ or $\frac{360°}{b}$

Phase Shift = h $(x - \pi/2)$ right $\pi/2$ $(x + \pi/4)$ left $\pi/4$

Vertical Displacement (new center line) = k (sinusoidal axis)

Ex. #1: A sine function is given by the equation $y = 3\sin 2(x - \frac{\pi}{4}) + 2$. Determine the following.

(a) Amplitude $= |3| = 3$

(b) Period $= \frac{2\pi}{2} = \pi$

(c) Phase shift

$\frac{\pi}{4}$ right

(d) Vertical displacement

2 up

(e) Domain

$\{x \mid x \in \mathbb{R}\}$

(f) Range

Max $= 1(3) + 2$
Max $= 5$

Min $= -1(3) + 2$
Min $= -1$

$\{y \mid -1 \le y \le 5,\ y \in \mathbb{R}\}$

(g) y-intercept $x = 0$

$y = 3\sin 2(0 - \frac{\pi}{4}) + 2$

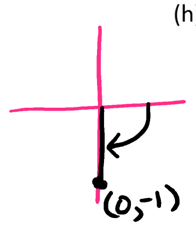
$y = 3\underbrace{\sin(-\tfrac{\pi}{2})}$ $+ 2$

$y = 3\left(\frac{-1}{1}\right) + 2$

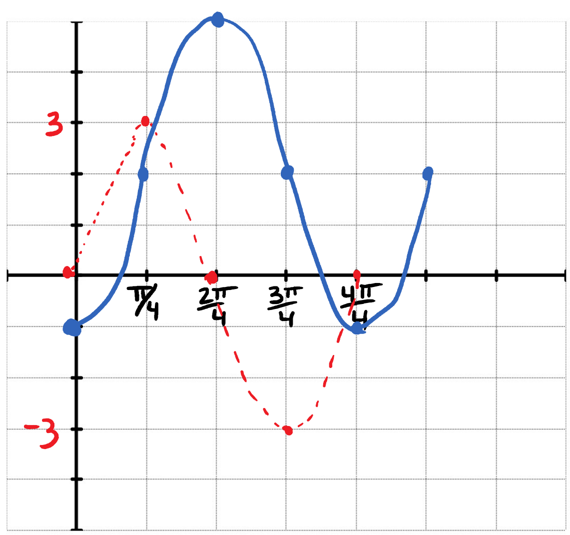
$y = -3 + 2$

$y = -1$

(h) Sketch the graph



$\sin\theta = \frac{y}{r}$



old

0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

$a = 3$
Mult y's by 3

$b = 2$
divide x's by 2

graph these points

0	0
$\pi/4$	3
$\frac{2\pi}{4} = \pi/2$	0
$3\pi/4$	-3
$\frac{4\pi}{4} = \pi$	0

phase shif $\pi/4$

Common denominator for phase shift and new x-values

Translate right $\pi/4$ (1 box)
up 2 (2 boxes)

Ex. #2: A cosine function is given by the equation $y=-2\cos\frac{2}{3}(x-\frac{\pi}{4})+1$. Determine the following.

(a) Amplitude $= |-2| = 2$

(b) Period $= \frac{2\pi}{2/3} = 2\pi \cdot \frac{3}{2} = 3\pi$

(c) Phase shift

$\frac{\pi}{4}$ right

(d) Vertical displacement

up 1

(e) Domain

$\{x \mid x \in \mathbb{R}\}$

(f) Range

Max $= 1(2)+1$

Max $= 3$

Min $= -1(2)+1$

Min $= -1$

$\{y \mid -1 \le y \le 3 \quad y \in \mathbb{R}\}$

(g) y-intercept $x = 0$

$y = -2\cos\frac{2}{3}(0-\pi/4)+1$

$y = -2\cos(-\frac{\pi}{6})+1$

$y = -2\left(\frac{\sqrt{3}}{2}\right)+1$

$y = -\sqrt{3}+1 \qquad y \approx -0.7$

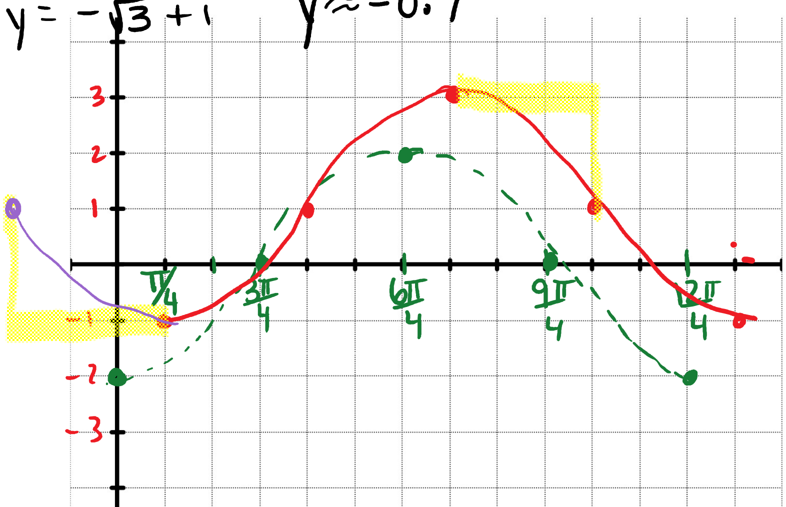
(h) Sketch the graph

old	
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

$a = -2$
Mult y's

$b = \frac{2}{3}$ Divide x's by $2/3$

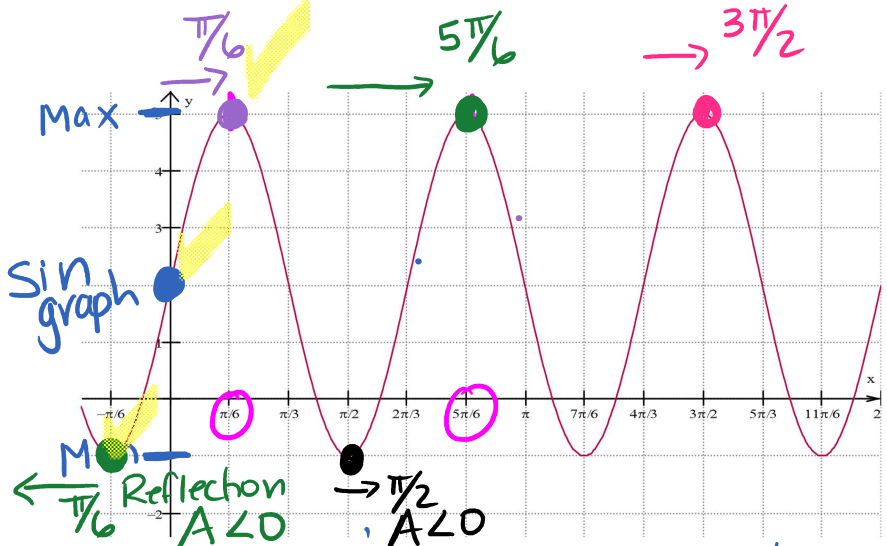
0	-2
$3\pi/4$	0
$6\pi/4 = 3\pi/2$	2
$9\pi/4$	0
$\frac{12\pi}{4} = 6\pi/2$	-2



Phase shift $\rightarrow \frac{\pi}{4}$ (1 box)

Vertical displacement $\uparrow 1$ (1 box).

Ex. #3: The partial graph of a cosine function is shown. Determine the equation of the function in the form $y = A\cos B(x + C) + D$



Amplitude

$$Amp = \frac{|Max - Min|}{2}$$

$$Amp = \frac{|5 - (-1)|}{2}$$

$$Amp = \frac{6}{2} = 3$$

$$A = 3$$

Vertical Displacement

$$VD = Max - Amp$$

$$VD = 5 - 3$$

$$VD = 2$$

$$D = 2$$

or

$$VD = Min + Amp$$

$$VD = -1 + 3$$

$$VD = 2$$

Period = subtract 2 x-value (peaks)

$$period = \frac{5\pi}{6} - \frac{\pi}{6}$$

$$period = \frac{4\pi}{6} = \frac{2\pi}{3}$$

$$period = \frac{2\pi}{B}$$

$$\frac{2\pi}{3} = \frac{2\pi}{B}$$

$$3 = B$$

phase shift

$\rightarrow \pi/6$

$$y = 3\cos 3\left(x - \frac{\pi}{6}\right) + 2$$

$\rightarrow 5\pi/6$

$$y = 3\cos 3\left(x - 5\pi/6\right) + 2$$

$\leftarrow \pi/6$

$$y = -3\cos 3\left(x + \pi/6\right) + 2$$