Pre-Calculus 12 Name: ______________________

Pre-Calculus 12: Final Review

Chapter 1 &2 Transformations

1. In what order should transformations be applied to a graph?

① transformations and reflections

② translations

2. Describe the transformations in each equation in an appropriate order.

a) $2y - 8 = 6f(x-2)$

$2y = 6f(x-2) + 8$

$y = 3f(x-2) + 4$

Vertical stretch factor of 3

Horizontal translation right 2

Vertical translation up 4

b) $y = -3f[-4(x-1)] + 2$

Reflection over x-axis

Vertical stretch factor of 3

Reflection over y-axis

Horizontal stretch factor $\frac{1}{4}$

Horizontal translation Right 1

Vertical translation up 2

3. Draw the transformations of each graph.

a) $y = f(-\frac{1}{4}x) + 1$

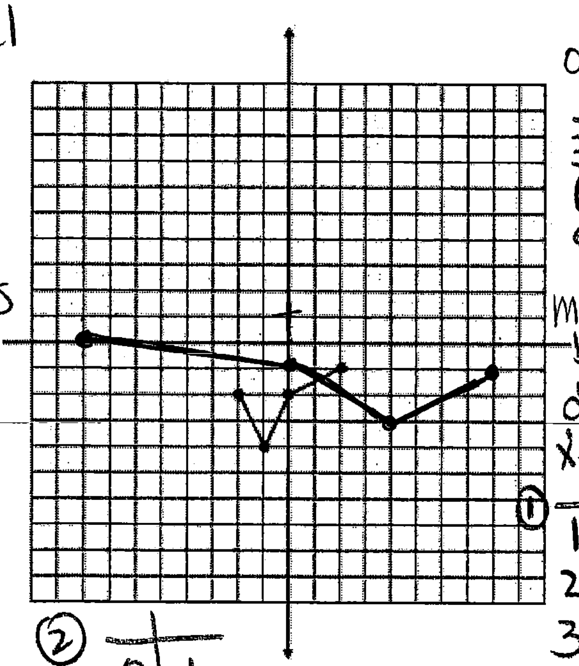
original

-2	-2
-1	-4
0	-2
2	-1

divide x's by (-1/4)

①

8	-2
4	-4
0	-2
-8	-1



add 1 to y's

②

8	-1
4	-3
0	-1
-8	0

b) $f(x) = 2f(3x-6) - 10$

$f(x) = 2f(3(x-2)) - 10$

original

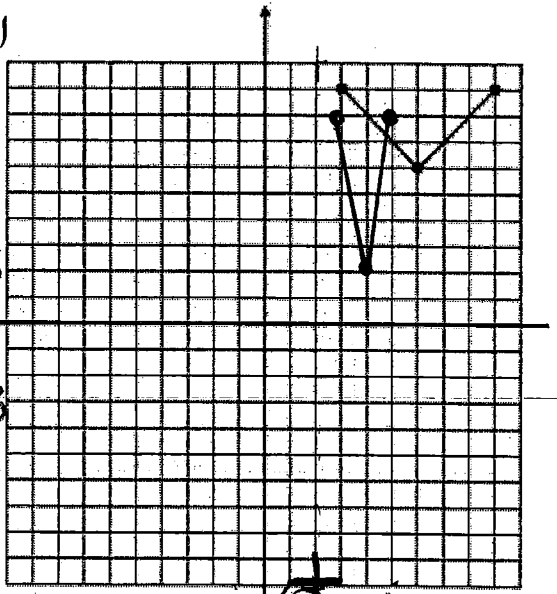
3	9
6	6
9	9

multy's by 2

divide xs by 3

①

1	18
2	12
3	18



add 2 to X's

subtract 10 from y's

②

3	8
4	2
5	8

4. The following transformations are applied to a function $y = f(x)$

Vertical stretch by a factor of 4 $a=4$

Horizontal stretch by a factor of 3 $b=\frac{1}{3}$

Reflection over the x-axis $a<0$

Translated 2 up, 5 to the left

$y=af(b(x-h)+k$

$y=-4f(\frac{1}{3}(x+5))+2$

a) Create a mapping notation for the transformations

$(x,y) \rightarrow (3x-5, -4y+2)$

b) If the point (-2,5) is on f(x), use the mapping notation to find the new point after the transformations are applied

-2	5

①

Mult y's by (-4)

Divide x's by $\frac{1}{3}$ or Mult by 3

-6	-20

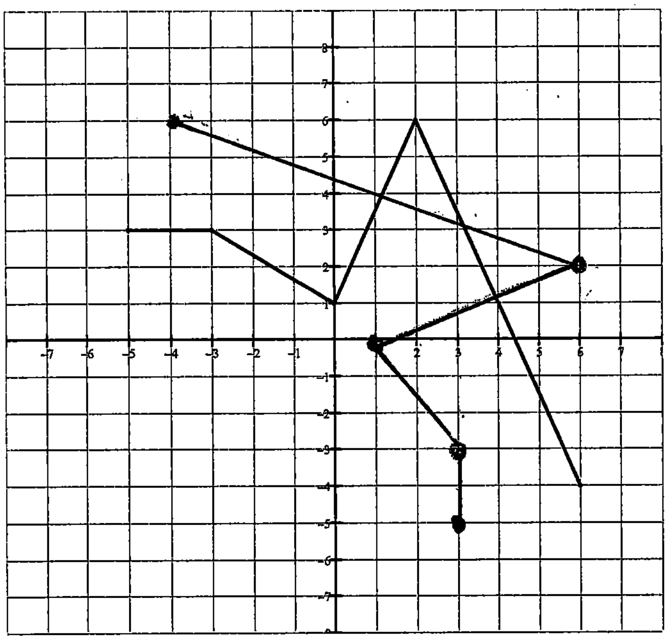
②

subtract 5 from x's

add 2 to y's

-11	-18

5. Sketch the inverse of the relation.



Original

x	y
-5	3
-3	3
0	1
2	6
6	-4

switch x and y

inverse

x	y
3	-5
3	-3
1	0
6	2
-4	6

6. Find the inverse of $f(x) = \frac{3}{x-2}$ Switch x and y, then solve for y

$y = \frac{3}{x-2}$

$x = \frac{3}{y-2}$

$(y-2)x = \left(\frac{3}{\cancel{(y-2)}}\right)\cancel{(y-2)}$

$\frac{(y-2)(x)}{\cancel{x}} = \frac{3}{\cancel{x}}$

$y - 2 = \frac{3}{x}$

$y = \frac{3}{x} + 2$

7. The domain and range of a function are: $\{x | -3 \leq x \leq 6 \quad x \in R\}$ $\{y | y > 7 \quad y \in R\}$ State the domain and range of the inverse.

Inverse $\{x | x > 7 \quad x \in \mathbb{R}\}$

$\{y | -3 \leq y \leq 6 \quad y \in \mathbb{R}\}$

8. Sketch the graph of the function using transformations. List the transformations in an appropriate order. State its domain and range.

$y = 2\sqrt{x-3} + 4$

orig

0	0
1	1
4	2
9	3

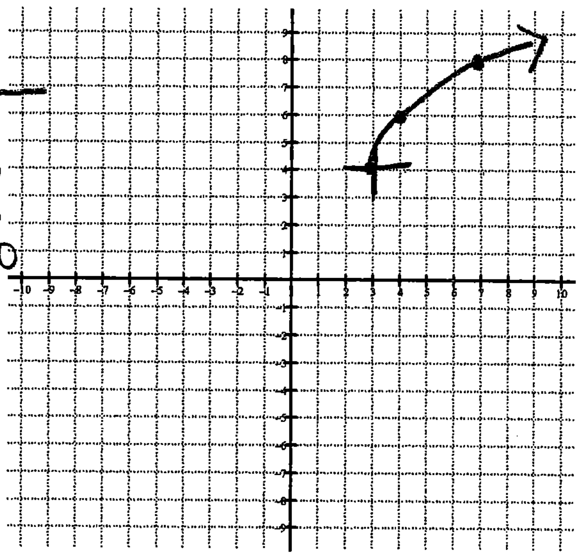
mult y's by 2

0	0
1	2
4	4
9	6

add 3 to x's

add 4 to y's

3	4
4	6
7	8
12	10



Domain: $\{x | x \geq 3 \quad x \in \mathbb{R}\}$

Range: $\{y | y \geq 4 \quad y \in \mathbb{R}\}$

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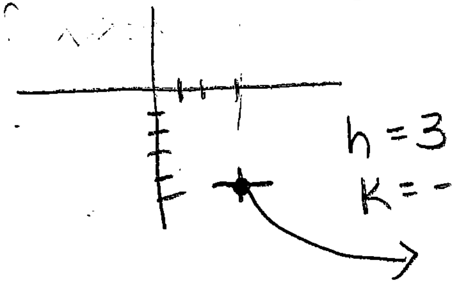
9. Write a **single equation** for a radical function with the given domain and range.

D: $\{x / x \geq 3, x \in \Re\}$

R: $\{y / y \leq -5, y \in \Re\}$



h = 3

k = -5

reflection over x-axis so a < 0

$y = -\sqrt{x-3} - 5$

10. Solve the equation graphically.

$2\sqrt{x+2} = 1 - x$

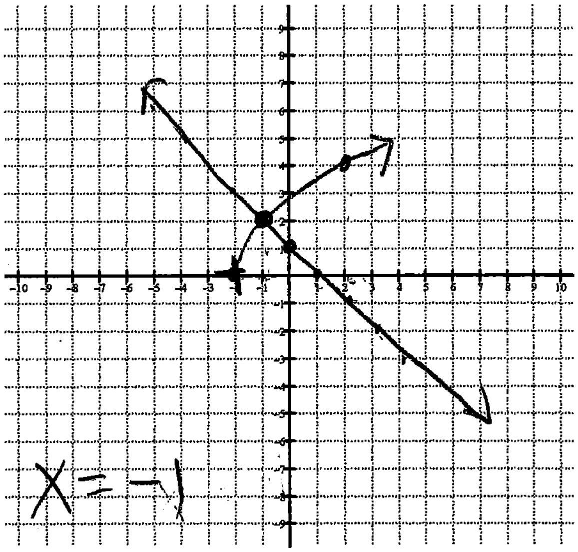
$y_1 = 2\sqrt{x+2}$ $y_2 = 1 - x$ or $y_2 = -x + 1$

$y = \sqrt{x}$

0	0
1	1
4	2

$y = 2\sqrt{x}$

0	0
1	2
4	4



$X = -1$

Chapter 3 Polynomials

1. For the following polynomial function, state the following:

$f(x) = x^4 - 5x^3 + 2x^2 + 20x - 24$

a) degree 4

b) type quartic

c) leading coefficient positive

d) constant term -24

e) the value of the y-intercept $(0, -24)$

f) maximum possible number of x-intercepts 4

g) end behavior of the corresponding graph Up into Quad 1 and up into Quad 2

2. Use the Factor Theorem to determine whether $x^4 - 2x^3 + 3x - 4$ has a factor of $x - 2$

$P(x) = x^4 - 2x^3 + 3x - 4$

$P(2) = 2^4 - 2(2)^3 + 3(2) - 4$

$P(2) = 16 - 16 + 6 - 4$

$P(2) = 2$ (x-2) is NOT a factor

3. For the following function determine a) the x-intercepts, b) the degree c) end behavior of the graph, d) the zeroes and their multiplicity, e) the y-intercept of the graph, and f) the intervals where the function is positive and g) the intervals where the function is negative.

$$f(x) = x^4 + 4x^3 - 7x^2 - 34x - 24$$

y-intercept

$(0, -24)$

degree and end behavior

deg 4 leading coeff ⊕

up into Quad 1 and 2

x-intercepts

zeroes and multiplicity

$\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24\}$

$f(-1) = (-1)^4 + 4(-1)^3 - 7(-1)^2 - 34(-1) - 24$

$f(-1) = 1 - 4 - 7 + 34 - 24$

$f(-1) = 0$ $(x+1)$ is a factor

$$\begin{array}{r|rrrrr} +1 & 1 & 4 & -7 & -34 & -24 \\ & & 1 & 3 & -10 & -24 \\ \hline & 1 & 3 & -10 & -24 & 0 \end{array}$$

$f(x) = (x+1)\underbrace{(x^3 + 3x^2 - 10x - 24)}_{g(x)}$

$\rightarrow$ $g(-2) = (-2)^3 + 3(-2)^2 - 10(-2) - 24$

$g(-2) = -8 + 12 + 20 - 24$

$g(-2) = 0$

$(x+2)$ is a factor

$$\begin{array}{r|rrrr} +2 & 1 & 3 & -10 & -24 \\ & & 2 & 2 & -24 \\ \hline & 1 & 1 & -12 & 0 \end{array}$$

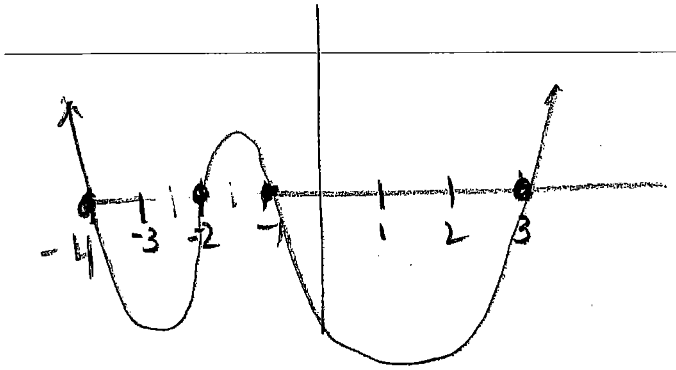
$f(x) = (x+1)(x+2)(x^2 + x - 12)$

$f(x) = (x+1)(x+2)(x+4)(x-3)$

Zeroes $x = -1$, $x = -2$ $x = -4$ $x = 3$

All Multiplicity 1

intervals of positive and negative



positive $x < -4$, $x > 3$

$-2 < x < -1$

Negative $-4 < x < -2$

$-1 < x < 3$

4. Find the value of “k” if the remainder is 3 when $x^3 - x^2 + kx - 15$ is divided by $x - 2$.

$f(x) = x^3 - x^2 + Kx - 15$ $R = 3$

$f(2) = 3$

$3 = 2^3 - 2^2 + K(2) - 15$

$3 = 8 - 4 + 2K - 15$

$3 = 2K - 11$

$14 = 2K$ $K = 7$

Chapter 4 Trigonometry and the Unit Circle

1. Change the given angle from radians to degrees or vice-versa.

a) $\frac{5\pi}{9}$

$\frac{5\cancel{\pi}}{\cancel{9}} \times \frac{\overset{20}{\cancel{180}}}{\cancel{\pi}}$

$= 100°$

b) 240°

$24\cancel{0}° \times \frac{\pi}{18\cancel{0}°}$

$= \frac{\overset{4}{\cancel{24}}\pi}{\cancel{18}_3}$

$= 4\pi/3$

2. Find one positive and one negative co-terminal angle for the original angles in question #1.

$= \frac{5\pi}{9} + 2\pi$

$= \frac{5\pi}{9} + \frac{18\pi}{9}$

$= \frac{23\pi}{9}$

$= \frac{5\pi}{9} - 2\pi$

$= \frac{5\pi}{9} - \frac{18\pi}{9}$

$= \frac{-13\pi}{9}$

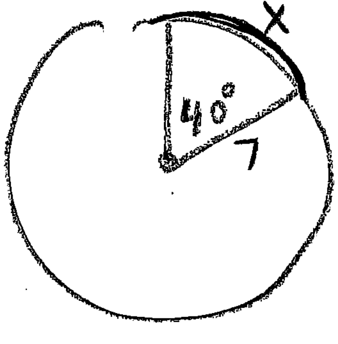
$= 240° + 360°$

$= 600°$

$= 240° - 360°$

$= -120°$

3. A circle as central angle of 40° and a radius of 7ft. Find the arclength of the sector.

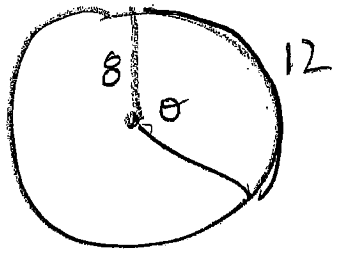


$\frac{40°}{360°} = \frac{x}{2\pi(7)}$

$x = \frac{\overset{1}{\cancel{40°}} \cdot 2\pi(7)}{\cancel{360°}_9}$

$x = \frac{14\pi}{9}$ or 4.89

4. A radius of a circle is 8 cm, and the length of an arc on the circle is 12 cm. In radians, what is the central angle that subtends this arc length?



$$\frac{\theta}{2\pi} = \frac{12}{2\pi(8)}$$

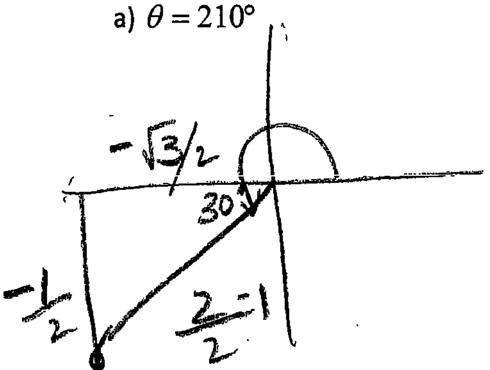
$$\theta = \frac{2\pi(12)}{2\pi(8)}$$

$$\theta = \frac{12}{8}$$

$$\theta = 3/2 \text{ rad}$$

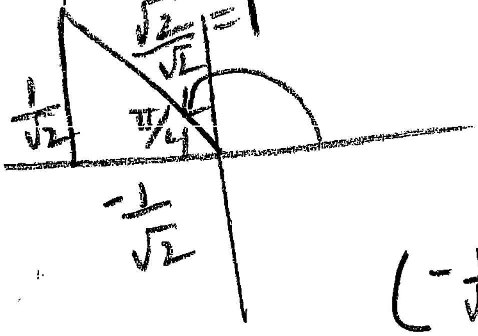
5. The point P(x,y) is located where the terminal arm of angle θ and the unit circle intersect. Determine the coordinates of point P if :

a) $\theta = 210°$



$$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

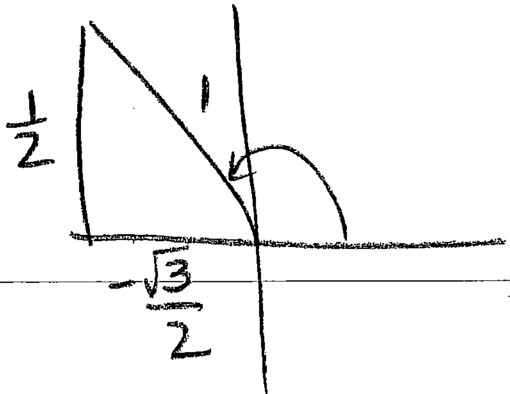
b) $\theta = \frac{3\pi}{4}$



$$\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

6. Identify a measure for the central angle θ in the interval $0 \le \theta \le 2\pi$ such that $P(\theta)$ is the given point.

a) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

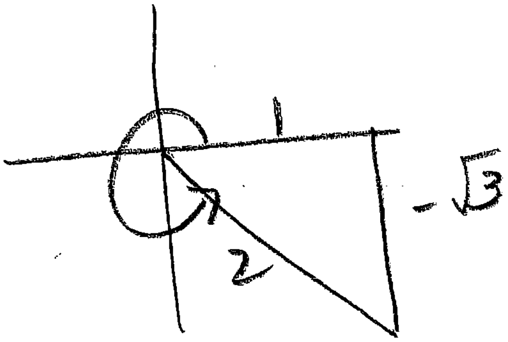


ref ∠ = $\pi/6$

$$\theta = \pi - \frac{\pi}{6}$$

$$\theta = 5\pi/6$$

b) $\left(1, -\sqrt{3}\right)$



ref ∠ = $\frac{\pi}{3}$

$$\theta = 2\pi - \frac{\pi}{3}$$

$$\theta = \frac{5\pi}{3}$$

Exact Values

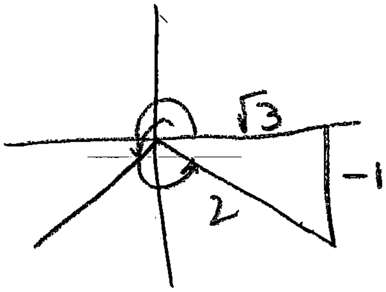
7. Solve algebraically for the domain stated. $0 \leq x < 2\pi$. ~~Give answers accurate to two deci~~mal ~~places~~

5 sin θ + 2 = 1 + 3 sin θ

$2\sin\theta = -1$

$\sin\theta = -\frac{1}{2}$

ref ∠ = $\pi/6$



$\theta = \pi + \frac{\pi}{6}$

$\theta = \frac{7\pi}{6}$

$\theta = 2\pi - \frac{\pi}{6}$

$\theta = \frac{11\pi}{6}$

Chapter 5 Trigonometric Functions and Graphs

1. Determine the key features for the function $y = -5sin\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right) + 15$

a) Amplitude: 5

b) Period: $2\pi/\frac{1}{2} = 4\pi$

c) Phase Shift: right $\pi/2$

d) Vertical displacement: up 15

e) Domain: $\{x \mid x \in \mathbb{R}\}$

f) Range: $\{y \mid 10 \leq y \leq 20 \ y \in \mathbb{R}\}$

Max = 1(5) + 15

Max = 20

Min = -1(5) + 15

Min = 10

2. Write the equation of each sine function in the form $y = asinb(x - c) + d$ given its characteristics.

a) amplitude 2, period π, phase shift $\frac{\pi}{3}$ to the left, vertical displacement 1 unit down

period $= \frac{2\pi}{b}$

$\pi = \frac{2\pi}{b}$

$b = 2$

$y = 2\sin 2(x + \pi/3) - 1$

b) amplitude $\frac{1}{4}$, period 6π, phase shift π to the right, vertical displacement 2 units up.

period $= \frac{2\pi}{b}$

$6\pi = \frac{2\pi}{b}$

$6\pi b = 2\pi$

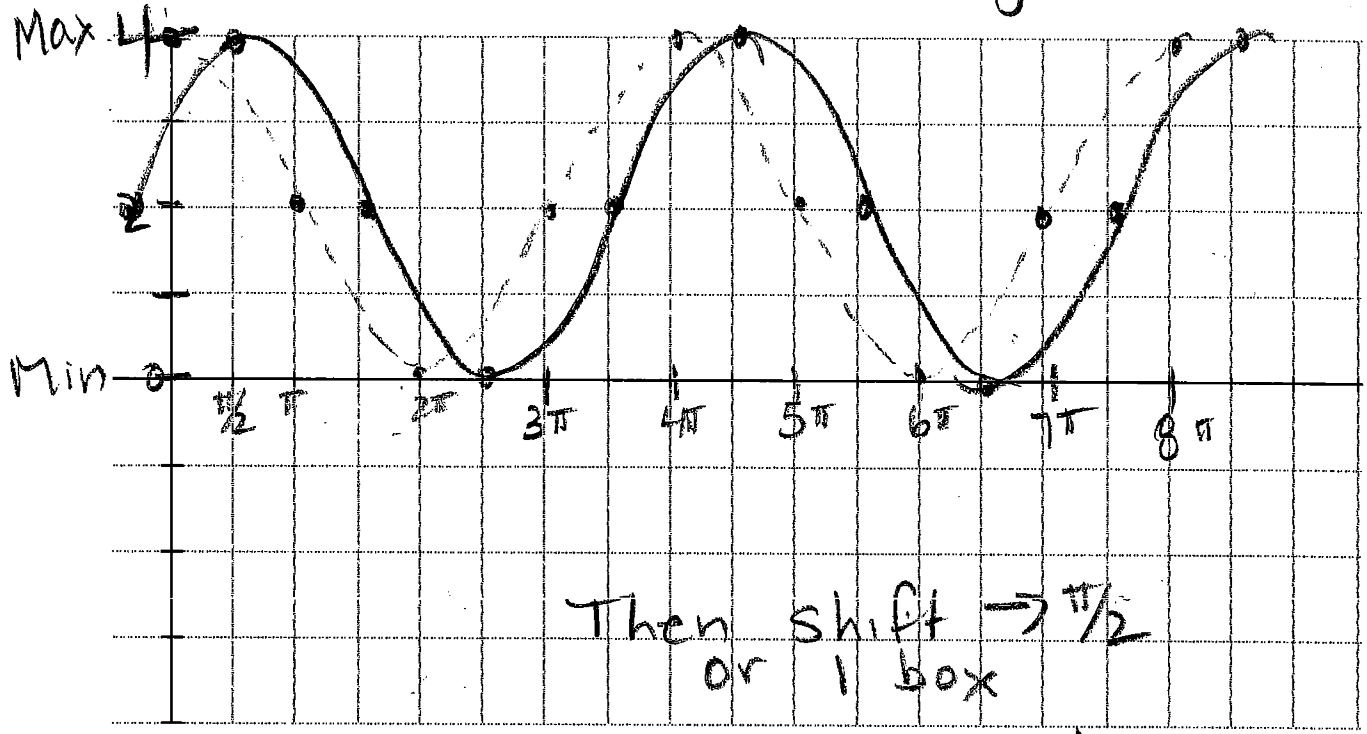
$b = \frac{2\pi}{6\pi} = \frac{1}{3}$

$y = \frac{1}{4}\sin\frac{1}{3}(x - \pi) + 2$

3. Graph the following function (show 2 periods) State the period and phase shift

$y = 2cos\frac{1}{2}\left(x - \frac{\pi}{2}\right) + 2$

period : $2\pi / \frac{1}{2} = 4\pi$ phase shift: right $\pi/2$



Then shift → π/2

or 1 box

0	1
π/2	0
π	-1
3π/2	0
2π	1

$b = \frac{1}{2}$

divide x's by $\frac{1}{2}$

0	1	Max
π	0	Mid
2π	-1	Min
3π	0	Mid
4π	1	Max

y's

Max = 1(2)+2 = 4

Min = -1(2)+2 = 0

Middle = 2

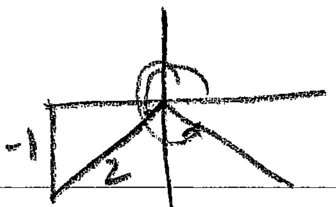
4. Solve the following trigonometric equations algebraically, using exact values. Show all work.

a) $4\sin(x - \frac{\pi}{3}) = -2 \qquad 0 \le x < 2\pi$

$\theta = x - \pi/3$

$4 \sin\theta = -2$

$\sin\theta = \frac{-1}{2}$



ref ∠ = π/6

$\theta_1 = \frac{7\pi}{6} \quad \theta_2 = \frac{11\pi}{6}$

$\frac{7\pi}{6} = x - \frac{\pi}{3}$

$\frac{7\pi}{6} + \frac{2\pi}{6} = x$

$\frac{9\pi}{6} = x$

$\frac{3\pi}{2} = x$

$\frac{11\pi}{6} = x - \frac{\pi}{3}$

$\frac{11\pi}{6} + \frac{2\pi}{6} = x$

$\frac{13\pi}{6} = x$

too big

$x = \frac{13\pi}{6} - 2\pi$

$x = \pi/6$

b) $2sin^2x + 5\sin x - 3 = 0 \qquad 0 \le x < 2\pi$

$m = \sin x$

$2m^2 + 5m - 3 = 0$

$\underline{\quad} \times \underline{\quad} = -6$

$\underline{6} + \underline{-1} = 5$

$2m^2 + 6m - 1m - 3 = 0$

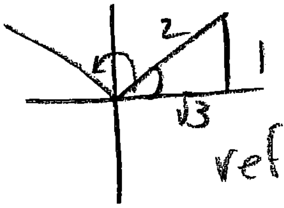
$2m(m+3) - 1(m+3) = 0$

$(m+3)(2m-1) = 0$

$m = -3 \qquad m = \frac{1}{2}$

$\sin x = -3 \qquad \sin x = \frac{1}{2}$

No soln



ref $\angle = \frac{\pi}{6}$

$x = \frac{\pi}{6} \qquad x = \pi - \frac{\pi}{6}$

$x = \frac{5\pi}{6}$

Chapter 6 Trigonometric Functions and Identities

1. Simplify the following:

a) $\cos(\alpha + 90°)$

$= \cos\alpha\cos 90° - \sin\alpha\sin 90°$

$= \cos\alpha(0) - \sin\alpha(1)$

$= -\sin\alpha$

b) $sin25°cos65° + cos25°sin65°$

sum for sin

$= \sin(25 + 65)$

$= \sin 90°$

$= 1$

2. Solve the following, accurate to 2 decimal places for $0 \le \theta < 2\pi$.

a) $2sec^2x + 5secx - 3 = 0 \qquad m = \sec x$

$2m^2 + 5m - 3 = 0$

$2m^2 + 6m - 1m - 3 = 0$

$2m(m+3) - 1(m+3) = 0$

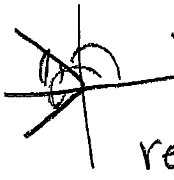
$(m+3)(2m-1) = 0$

$m = -3 \qquad m = \frac{1}{2}$

$\sec x = -3 \qquad \sec x = \frac{1}{2}$

$\cos x = -\frac{1}{3} \qquad \underbrace{\cos x = 2}$

No Soln



$x = \cos^{-1}(-\frac{1}{3})$

$x = 1.91$

ref $\angle \doteq 1.23$

$x = \pi + 1.23$

$x = 4.37$

b) $2cos^2x = -3sinx$

$2(1 - \sin^2 x) = -3\sin x$

$2 - 2\sin^2 x = -3\sin x$

$0 = 2\sin^2 x - 3\sin x - 2$

$m = \sin x$

$0 = 2m^2 - 3m - 2$

$0 = 2m^2 - 4m + 1m - 2$

$0 = 2m(m-2) + 1(m-2)$

$0 = (m-2)(2m+1)$

$m = 2 \qquad m = -\frac{1}{2}$

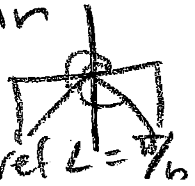
$\sin x = 2 \qquad \sin x = -\frac{1}{2}$

No Soln

$x = \sin^{-1}(-\frac{1}{2})$

$x \doteq 5.76$ or

$x = 3.66$



ref $\angle = \pi/6$

$x = 7\pi/6$

$x = 11\pi/6$

3. Solve for all possible solutions in radians. (Find a general solution)

$\underbrace{\sin 2x}\, = 2\sin x$

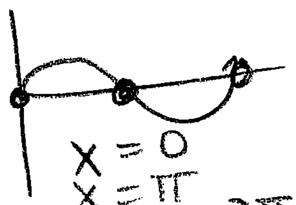
$2\sin x\cos x = 2\sin x$

$2\sin x\cos x - 2\sin x = 0$

$2\sin x(\cos x - 1) = 0$

$2\sin x = 0$

$\sin x = 0$



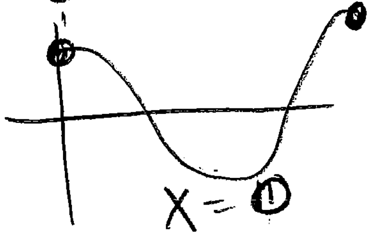
$x = 0$

$x = \pi$

$x = 2\pi$

$\cos x - 1 = 0$

$\cos x = 1$



$x = 0$

$x = 2\pi$

$x = n\pi \quad n \in I$

4. Use sum or difference identities to find the exact value of each trigonometric expression.

a) $sin 15°$

$\sin(45° - 30°)$

$= \sin 45° \cos 30° - \cos 45° \sin 30°$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$ or $\frac{\sqrt{6}-\sqrt{2}}{4}$

b) $tan 165°$

$\tan(135° + 30°)$

$= \frac{\tan 135° + \tan 30°}{1 - (\tan 135°)(\tan 30°)}$

$= \frac{-1 + \frac{1}{\sqrt{3}}}{1 + (-1)(\frac{1}{\sqrt{3}})}$

$= \frac{-1 + \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$

$= \frac{-\sqrt{3} + 1}{\sqrt{3} + 1}$

$= \frac{-\sqrt{3}+1}{\sqrt{3}+1} \cdot \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{-3 + \sqrt{3} + \sqrt{3} - 1}{3 - \sqrt{3} + \sqrt{3} - 1}$

$= \frac{-4 + 2\sqrt{3}}{2}$

$= -2 + \sqrt{3}$

5. Simplify the following:

a) $cot^2x\ sin^2x + cos^2x$

$\frac{\cos^2 x}{\cancel{\sin^2 x}} \cdot \cancel{\sin^2 x} + \cos^2 x$

$\cos^2 x + \cos^2 x$

$2\cos^2 x$

b) $\frac{sec\theta - cos\theta}{csc\theta - sin\theta}$

$= \frac{\frac{1}{\cos\theta} - \frac{\cos\theta}{1}}{\frac{1}{\sin\theta} - \frac{\sin\theta}{1}}$

$= \frac{\frac{1-\cos^2\theta}{\cos\theta}}{\frac{1-\sin^2\theta}{\sin\theta}}$

$= \frac{1-\cos^2\theta}{\cos\theta} \cdot \frac{\sin\theta}{1-\sin^2\theta}$

$= \frac{\sin^2\theta}{\cos\theta} \cdot \frac{\sin\theta}{\cos^2\theta}$

$= \frac{\sin^3\theta}{\cos^3\theta}$

$= \tan^3\theta$

c) $(1+\cos\theta)(\csc\theta-\cot\theta)$

$\csc\theta - \cot\theta + \cos\theta\csc\theta - \cos\theta\cot\theta$

$\csc\theta - \cot\theta + \cos\theta \cdot \frac{1}{\sin\theta} - \cos\theta \cdot \frac{\cos\theta}{\sin\theta}$

$\csc\theta - \cot\theta + \cot\theta - \frac{\cos^2\theta}{\sin\theta}$

$\frac{1}{\sin\theta} - \frac{\cos^2\theta}{\sin\theta}$

$\frac{1-\cos^2\theta}{\sin\theta} = \frac{\sin^2\theta}{\sin\theta} = \sin\theta$

6. Prove the identity.

a) $\sin^3 x + \sin x\cos^2 x = \sin x$

$\sin x(\sin^2 x + \cos^2 x) = RS$

$\sin x(1) = R.S$

$\sin x = \sin x$

b) $\frac{1+\cos x+\cos 2x}{\sin x+\sin 2x} = \cot x$

$\frac{1+\cos x+2\cos^2 x-1}{\sin x+2\sin x\cos x} = \cot x$

$\frac{\cos x+2\cos^2 x}{\sin x(1+2\cos x)} = R.S$

$\frac{\cos x(1+2\cos x)}{\sin x(\cancel{1+2\cos x})} = R.S$

$\frac{\cos x}{\sin x} = R.S$

$\cot x = \cot x$

c) $\frac{\sin 2x}{2-2\cos^2 x} = \cot x$

$\frac{2\sin x\cos x}{2(1-\cos^2 x)} = R.S$

$\frac{\cancel{2}\cancel{\sin x}\cos x}{\cancel{2}\sin^2 x} = R.S$

$\frac{\cos x}{\sin x} = R.S$

$\cot x = \cot x$

d) $\frac{\cot x}{\csc x-1} = \frac{\csc x+1}{\cot x}$

$\frac{\cot x\cdot(\csc x+1)}{(\csc x-1)(\csc x+1)} = R.S$

$\frac{\cot x(\csc x+1)}{\csc^2 x+\csc x-\csc x-1} = R.S$

$\frac{\cot x(\csc x+1)}{\csc^2 x-1} = R.S$

$\frac{\cot x(\csc x+1)}{\cot^2 x}$

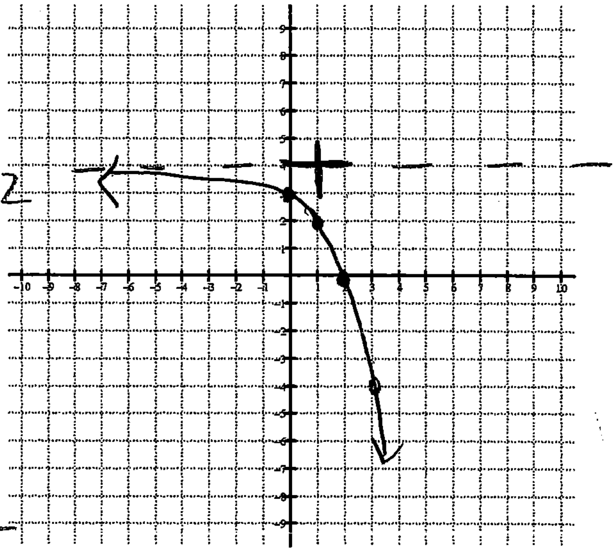
$\csc x+1 = \frac{\csc x+1}{\cot x}$

Chapter 7 Exponential Functions

1. Sketch the graph of each function using transformations and tables of values. List the transformations in an appropriate order. (4 marks each)

(a) $y = -2(2)^{x-1} + 4$

- Reflection over x-axis
- Vertical Stretch factor 2
- Horizontal Translation Right 1
- Vertical translation up 4



base function

$y = 2^x$

-1	1/2
0	1
1	2
2	4
3	8

$a = -2$, mult y's by (-2)

-1	-1
0	-2
1	-4
2	-8
3	-16

too big

2. Solve

a) $64^{4x} = 16^{(x+5)}$

$(4^3)^{4x} = (4^2)^{(x+5)}$

$4^{12x} = 4^{2x+10}$

$12x = 2x + 10$

$10x = 10$

$x = 1$

b) $36^{-3n} \cdot 216 = \left(\frac{1}{216}\right)^{-2n}$

$(6^2)^{-3n} \cdot 6^3 = (6^{-3})^{-2n}$

$6^{-6n} \cdot 6^3 = 6^{6n}$

$6^{-6n+3} = 6^{6n}$

$-6n + 3 = 6n$

$3 = 12n$

$\frac{1}{4} = n$

c) $\frac{9^{3x}}{243^{-x-1}} = 81^{2x}$

$$\frac{(3^2)^{3x}}{(3^5)^{-x-1}} = (3^4)^{2x}$$

$$\frac{3^{6x}}{3^{-5x-5}} = 3^{8x}$$

$$3^{11x+5} = 3^{8x}$$

$$11x + 5 = 8x$$

$$5 = -3x$$

$$\frac{-5}{3} = x$$

3. The half –life of sodium-24 is 17 hours. A chemistry teacher has 40 mg of sodium-24. After how long will only 5 mg remain?

$$A = A_0(c)^{t/T}$$

$c = \frac{1}{2}$

$T = 17$

$A_0 = 40$

$A = 5$

$$\frac{5}{40} = \frac{40(\frac{1}{2})^{t/17}}{40}$$

$$\frac{1}{8} = \left(\frac{1}{2}\right)^{t/17}$$

$$\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{t/17}$$

$$3 = \frac{t}{17}$$

$$51 = t$$

4. A bacteria culture contains 6250 bacteria and doubles every 3 hours. What was the population 9 hours ago?

$$A = A_0(c)^{t/T}$$

$A_0 = 6250$

$c = 2$

$T = 3$

$t = -9$

$A =$

$$A = 6250(2)^{-9/3}$$

$$A = 6250(2)^{-3}$$

$$A = 6250\left(\frac{1}{8}\right)$$

$$A = 781.25$$

$$A \approx 781$$

5. The initial count was 530 bacteria in a culture. Ten hours later, there were 14310 bacteria. What is the tripling period for this type of bacteria?

$A_0 = 530$

$t = 10$

$A = 14310$

$c = 3$

$T = ?$

$$A = A_0(c)^{t/T}$$

$$\frac{14310}{530} = \frac{530(3)^{10/T}}{530}$$

$$27 = 3^{10/T}$$

$$3^3 = 3^{10/T}$$

$$3 = \frac{10}{T}$$

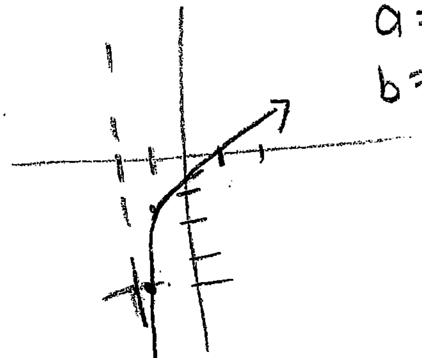
$$3T = 10$$

$$T = 10/3$$

$T = 3.33$ hrs

$5^y = x$

$1/5$	-1
1	0
5	1



$a = 3$
$b = 6$

$1/30$	-3
$\frac{1}{6}$	0
$5/6$	3

Chapter 8 Logarithmic Functions

1. For the equation $y = 3\log_5(6(x+2)) - 4$, state:

a) domain

$6(x+2) > 0$
$x + 2 > 0$
$x > -2$

$\{x \mid x > -2\ x \in \mathbb{R}\}$

b) range

$\{y \mid y \in \mathbb{R}\}$

c) equation of the asymptote

$x = -2$

d) x-intercept (if it exists)

$y = 0$
$0 = 3\log_5(6(x+2)) - 4$
$4 = 3\log_5(6(x+2))$
$\frac{4}{3} = \log_5(6(x+2))$
$5^{4/3} = 6x + 12$

$\frac{5^{4/3} - 12}{6} = x$

$x = -0.58$

e) y-intercept (if it exists)

$x = 0$
$y = 3\log_5(6(0+2)) - 4$
$y = 3\log_5 12 - 4$
$y = 3\frac{\log 12}{\log 5} - 4$
$y = 0.63$

2. Simplify to a single log and then evaluate if possible.

a) $2\log_2 12 - (\log_2 6 + \frac{1}{3}\log_2 27)$

$\log_2 12^2 - \log_2 6 - \log_2 27^{1/3}$
$\log_2 144 - \log_2 6 - \log_2 3$
$\log_2 \frac{144}{6 \cdot 3}$
$\log_2 8 = \log_2 2^3 = 3$

b) $2\log_5 4 + \log_5 3 - \log_5 11$

$\log_5 4^2 + \log_5 3 - \log_5 11$
$\log_5 \frac{16 \cdot 3}{11}$
$\log_5 \frac{48}{11}$

c) $\log x - 3\log y + \frac{2}{3}\log z$ $^{2/3}$

$\log x - \log y^3 + \log z^{2/3}$

$\log \frac{x z^{2/3}}{y^3}$

d) $\log_2(x+2) + \log_4 x$

↑ change to base 2

$\log_2(x+2) + \frac{\log_2 x}{\log_2 4}$

$\log_2(x+2) + \frac{\log_2 x}{2}$

$\log_2(x+2) + \frac{1}{2}\log_2 x$

$\log_2(x+2) + \log_2 x^{\frac{1}{2}}$

$\log_2\left[(x+2)\cdot x^{1/2}\right]$

$\log_2\left(x^{3/2} + 2x^{1/2}\right)$

3. Solve, answer to nearest hundredth if necessary.

a) $\log_7(2x-3) - \log_7(x+2) = 1$

$\log_7 \frac{2x-3}{x+2} = \log_7 7$

$\frac{2x-3}{x+2} = 7$

$2x-3 = 7(x+2)$

$2x-3 = 7x+14$

$-17 = 5x$

$x = -17/5$ ✗ No solution

$2x-3 > 0$

$2x > 3$

$x > 3/2$

$x+2 > 0$

$x > -2$

b) $\log_b(x+2) - \log_b 4 = \log_b 3x$

$\log_b \frac{x+2}{4} = \log_b 3x$

$x+2 > 0$

$x > -2$ ✓

$3x > 0$

$x > 0$ ✓

$\frac{x+2}{4} = 3x$

$x+2 = 12x$

$2 = 11x$

$\boxed{2/11 = x}$ ✓

$x+4>0$ $x+12>0$

$x>-4$ $x>-12$

c) $2\log_4(x+4)-\log_4(x+12)=1$

$\log_4(x+4)^2-\log_4(x+12)=\log_4 4$

$\log_4\frac{(x+4)^2}{x+12}=\log_4 4$

$\frac{(x+4)^2}{x+12}=4$

$x^2+8x+16=4(x+12)$

$x^2+8x+16=4x+48$

$x^2+4x-32=0$

$(x+8)(x-4)=0$

~~$x=-8$~~ $\boxed{x=4}$ ✓

d) $\frac{2\ln(5x-2)}{2}=\frac{16}{2}$

$\ln(5x-2)=8$

$e^8=5x-2$

$e^8+2=5x$

$\frac{e^8+2}{5}=x$

$x=596.6$

4. Solve, answer to nearest hundredth if necessary.

a) $9^{2x-1}=71^{x+2}$

$\log 9^{2x-1}=\log 71^{x+2}$

$(2x-1)\log 9=(x+2)\log 71$

$2x\log 9-\log 9=x\log 71+2\log 71$

$2x\log 9-x\log 71=2\log 71+\log 9$

$x(2\log 9-\log 71)=2\log 71+\log 9$

$x=\frac{2\log 71+\log 9}{2\log 9-\log 71}$ $x=81.37$

b) $4(7^{x+2})=9^{2x-3}$

$\log[4\cdot 7^{x+2}]=\log 9^{2x-3}$

$\log 4+\log 7^{x+2}=\log 9^{2x-3}$

$\log 4+(x+2)\log 7=(2x-3)\log 9$

$\log 4+x\log 7+2\log 7=2x\log 9-3\log 9$

$\log 4+2\log 7+3\log 9=2x\log 9-x\log 7$

$\log 4+2\log 7+3\log 9=x(2\log 9-\log 7)$

$\frac{\log 4+2\log 7+3\log 9}{2\log 9-\log 7}=x$

$x=4.85$

c) $e^{3x+1}=2$

$\ln e^{3x+1}=\ln 2$

$(3x+1)\ln e=\ln 2$

$(3x+1)(1)=\ln 2$

$3x+1=\ln 2$

$3x=\ln 2-1$

$x=\frac{\ln 2-1}{}$ $x=-0.102$

Chapter 9 Rational Functions

1. For each function, find the locations of any vertical asymptotes, points of discontinuity, and intercepts.

a) $y = \frac{x^2+4x}{x^2+9x+20}$

$y = \frac{x\cancel{(x+4)}}{\cancel{(x+4)}(x+5)} = \frac{x}{x+5}$

Vertical asymptote $x=-5$

point of discontinuity

$x=-4 \quad y = \frac{-4}{-4+5} = \frac{-4}{1}$

$(-4,-4)$

y-int

$y = \frac{0}{0+5}$

$y = \frac{0}{5}$

$y = 0 \quad (0,0)$

x-int

$0 = \frac{x}{x+5}$

$(x+5)(0) = x$

$0 = x$

$(0,0)$

b) $y = \frac{2x^2-5x-3}{x^2-1}$

$\frac{x}{-6} \overset{+}{} \frac{I}{} \quad = -6 \quad = -5$

$2x^2-6x+1x-3$

$2x(x-3)+1(x-3)$

$(x-3)(2x+1)$

$y = \frac{(x-3)(2x+1)}{(x-1)(x+1)}$

Vertical asymptote $x=1$ and $x=-1$

No point of discontinuity

y-int

$y = \frac{2(0)^2-5(0)-3}{0^2-1}$

$y = \frac{-3}{-1}$

$y = 3$

$(0,3)$

x-int

$0 = \frac{2x^2-5x-3}{x^2-1}$

$0 = 2x^2-5x-3$

$0 = (x-3)(2x+1)$

$x=3 \quad x=-\frac{1}{2}$

$(3,0) \quad (-\frac{1}{2},0)$

2. Graph the functions

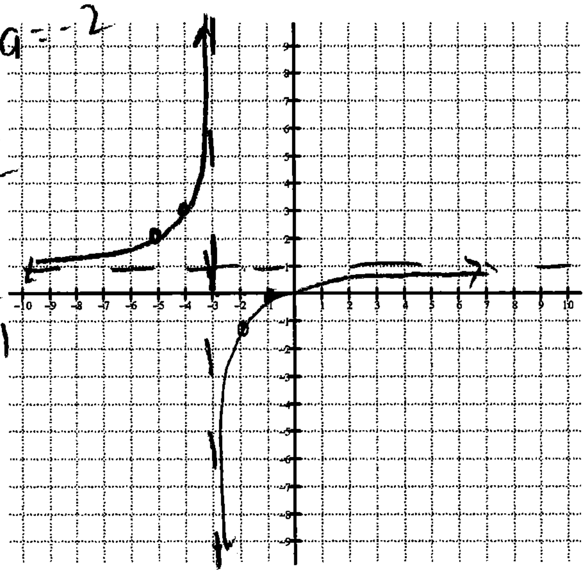
a) $y = \frac{-2}{x+3} + 1$

$\xleftarrow{}3 \quad \uparrow 1$

1	1

$a=-2$

1	-2
-2	1
-1	2
2	-1



b) $y = \frac{4x-5}{x-2}$

$$\begin{array}{r} 4 \\ x-2 \enclose{longdiv}{4x-5} \\ \underline{4x-8} \\ 3 \end{array}$$

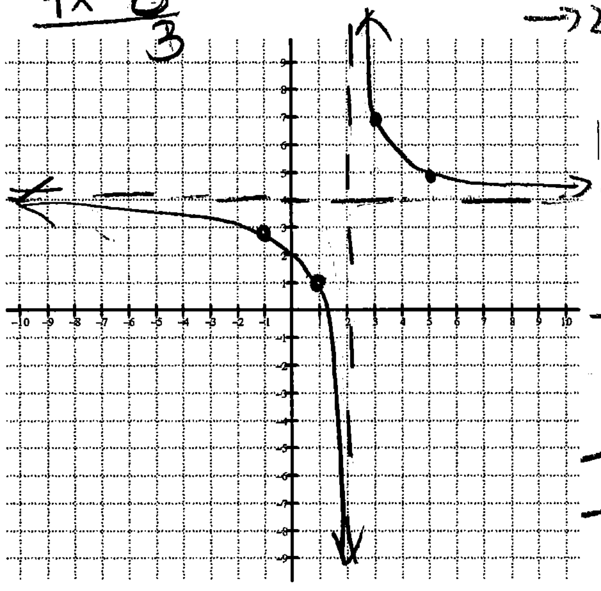
$y = \frac{3}{x-2} + 4$

$\to 2 \quad \uparrow 4$

$a=3$

1	1

1	3
3	1
-1	-3
-3	-1



Chapter 10 Composite Functions

1. If $f(x) = \sqrt{x+2}$ and $g(x) = |2x|$, find $f \circ g(-7)$

$f(g(-7))$

$g(-7) = |2(-7)|$

$g(-7) = |-14|$

$g(-7) = 14$

$f(g(-7))$

$= f(14)$

$= \sqrt{14+2}$

$= \sqrt{16}$

$= 4$

2. $f(x) = x^2 + 7$ and $g(x) = 2x - 1$ find $f(g(x))$

$f(g(x))$ $f(2x-1)$

$= (2x-1)^2 + 7$

$= 4x^2 - 4x + 1 \quad + 7$

$= 4x^2 - 4x + 8$

Chapter 12 Series

1. How many terms are in the sequence 2, 6, 18, …, 486

$a = 2$

$r = 6/2 = 3$

$t_n = 486$

$t_n = a \cdot r^{n-1}$

$486 = 2(3)^{n-1}$

$243 = 3^{n-1}$

$3^5 = 3^{n-1}$

$5 = n-1$

$6 = n$

2. The sum of an infinite geometric series is 63 and the first term is 21. Find the common ratio.

$S = 63$

$a = 21$

$S = \dfrac{a}{1-r}$

$63 = \dfrac{21}{1-r}$

$63(1-r) = 21$

$63 - 63r = 21$

$-63r = -42$

$r = \dfrac{-42}{-63}$

$r = \dfrac{2}{3}$

---

3. Find the sum of the first 12 terms of the series. $12 + 4 + \frac{4}{3} \ldots$

$a = 12$

$r = \frac{4}{12} = \frac{1}{3}$

$n = 12$

$S_{12} = \dfrac{12\left(\left(\frac{1}{3}\right)^{12} - 1\right)}{\frac{1}{3} - 1}$

$S_{12} = \dfrac{12\left(\left(\frac{1}{3}\right)^{12} - 1\right)}{-2/3} = 17.99966. \approx 18$