Optimization Review

Tuesday, December 6, 2016 10:37 AM

AP Calculus
4.6 Review

Optimization Review

1. Find two positive numbers such that their product is 192 and their sum is a minimum.

$S = x + y$

$xy = 192$

$x = \frac{192}{y}$

$S = \frac{192}{y} + y$

$S = \frac{192 + y^2}{y}$

$y > 0$

$S' = \frac{y(2y) - (192 + y^2)(1)}{y^2}$

$S' = \frac{2y^2 - 192 - y^2}{y^2}$

$S' = \frac{y^2 - 192}{y^2}$

$0 = y^2 - 192$ $y^2 = 0$

$y = \pm\sqrt{192}$ $y = 0$

$(0, \sqrt{192})(\sqrt{192}, \infty)$

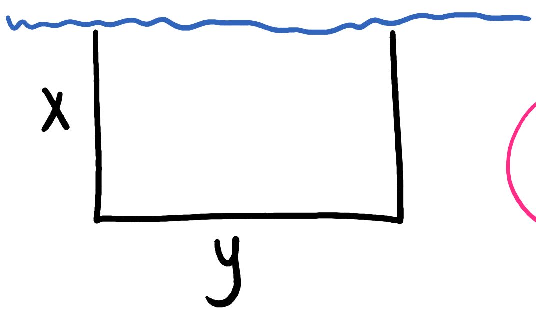
$y = 1$ $\ominus$ $y = 20$ $\oplus$

$y = \sqrt{192}$ Min

$x = \frac{192}{\sqrt{192}}$

$x = \sqrt{192}$

2. A farmer plans to fence in a rectangular pasture adjacent to a river. The pasture must contain 180,000 square meters in order to provide enough grass for the herd. What dimensions would require the least amount of fencing if no fencing is needed along the river?



$xy = 180\,000$

$y = \frac{180\,000}{x}$

$P = 2x + y$

$P = 2x + \frac{180\,000}{x}$

$P = \frac{2x^2 + 180\,000}{x}$

$x > 0$

$P' = \frac{x(4x) - (2x^2 + 180000)(1)}{x^2}$

$P' = \frac{4x^2 - 2x^2 - 180000}{x^2}$

$\boxed{P' = \frac{2x^2 - 180\,000}{x^2}}$

$2x^2 - 180\,000 = 0$ $x^2 = 0$

$x = \pm\sqrt{90000}$ $x = 0$

$x = \pm 300$

$(0, 300)(300, \infty)$

$x = 1$ $\ominus$ $x = 1000$ $\oplus$

$x = 300$ min

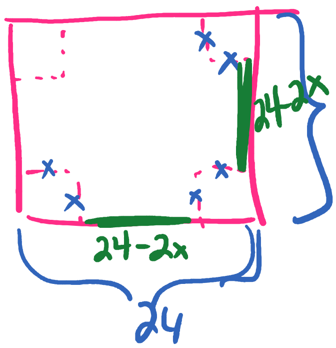
$y = \frac{180\,0\cancel{00}}{3\cancel{00}}$

$y = 600$

300 by 600 m

$x = \pm 300$

3. An open box is to be made from a square piece of material, 24 inches on a side, by cutting equal squares from each corner and turning up the sides. Find the maximum volume.



$0 < x < 12$

$V = x(24-2x)(24-2x)$

$V = x(24-2x)^2$

$V' = 1(24-2x)^2 + x(2)(24-2x)^1(-2)$

$V' = (24-2x)[24-2x-4x]$

$V' = (24-2x)(24-6x)$

$V' = 0$

$24-2x=0$ $\quad$ $24-6x=0$

$x=12$ $\quad$ $x=4$

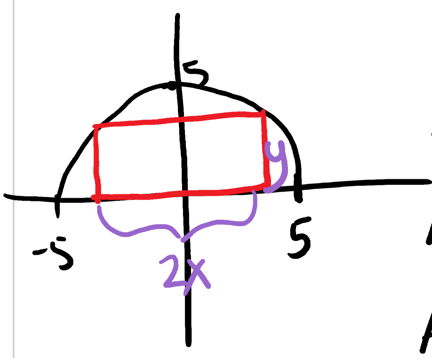
$V = 4(24-2(4))^2$

$V = 4(24-8)^2$

$V = 4(16)^2$

$V = 1024$ inches

4. A rectangle is bounded by the x-axis and the semi-circle $y = \sqrt{25 - x^2}$. What length and width should the rectangle have so that it produces a maximum area.



$0 < x < 5$

$A = 2xy \qquad y = \sqrt{25-x^2}$

$A = 2x(25-x^2)^{1/2}$

$A' = 2(25-x^2)^{1/2} + \cancel{2}x(\cancel{\tfrac{1}{2}})(25-x^2)^{-\frac{1}{2}}(-2x)$

$A' = (25-x^2)^{-\frac{1}{2}}\left[2(25-x^2)^1 - 2x^2\right]$

$A' = \dfrac{50-2x^2-2x^2}{(25-x^2)^{1/2}}$

$A' = \dfrac{50-4x^2}{(25-x^2)^{1/2}}$

$\left(0, \frac{5}{\sqrt{2}}\right)\left(\frac{5}{\sqrt{2}}, 5\right)$

$x=1 \qquad x=4$

$+ \qquad -$

Max $x = \frac{5}{\sqrt{2}}$

$50-4x^2=0$

$50 = 4x^2$

$50 = x^2$

$x = \pm \frac{5}{\sqrt{2}}$

$y = \sqrt{25 - \left(\frac{5}{\sqrt{2}}\right)^2}$

$y = \sqrt{25 - \frac{25}{2}}$

$50 = 4x^2$

$\frac{50}{4} = x^2$

$\frac{25}{2} = x^2$

$x = \pm \frac{5}{\sqrt{2}}$

$25 - x^2 = 0$

$x = \pm 5$

$y = \sqrt{25 - \frac{25}{2}}$

$y = \sqrt{\frac{50}{2} - \frac{25}{2}}$

$y = \sqrt{25/2} = \frac{5}{\sqrt{2}}$

rectangle $2x$ by y

$2\left(\frac{5}{\sqrt{2}}\right)$ by $5/\sqrt{2}$

$10/\sqrt{2}$ by $5/\sqrt{2}$