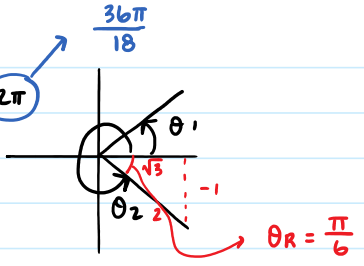
Assignment 5.4 - solutions

Wednesday, April 5, 2023 9:40 AM

1. $2\cos 3x = \sqrt{3}$ $\quad 0 \le x < 2\pi$

let $\theta = 3x$

$\cos\theta = \frac{\sqrt{3}}{2}$



$\theta_1 = \frac{\pi}{6}$ $\quad \theta_2 = \frac{11\pi}{6}$

period $= \frac{2\pi}{3} = \frac{12\pi}{18}$

$\theta = 3x$ so $x = \frac{\theta}{3}$

$x_1 = \frac{\pi}{6}\cdot\frac{1}{3}$ $\quad x_2 = \frac{11\pi}{6}\cdot\frac{1}{3}$

$x_1 = \frac{\pi}{18}$ $\quad x_2 = \frac{11\pi}{18}$

$x_3 = \frac{\pi}{18} + \frac{12\pi}{18}$ $\quad x_4 = \frac{11\pi}{18} + \frac{12\pi}{18}$ $\quad x_5 = \frac{13\pi}{18} + \frac{12\pi}{18}$ $\quad x_6 = \frac{23\pi}{18} + \frac{12\pi}{18}$

$x_3 = \frac{13\pi}{18}$ $\quad x_4 = \frac{23\pi}{18}$ $\quad x_5 = \frac{25\pi}{18}$ $\quad x_6 = \frac{35\pi}{18}$

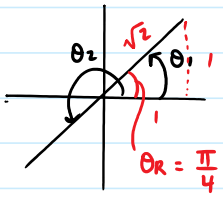
b) $5\tan\left(x - \frac{\pi}{3}\right) - 4 = 1$

let $\theta = x - \frac{\pi}{3}$

$5\tan\theta - 4 = 1$

$5\tan\theta = 5$

$\tan\theta = \frac{1}{1}$



$\theta_1 = \frac{\pi}{4}$ $\quad \theta_2 = \frac{5\pi}{4}$

$\theta = x - \frac{\pi}{3}$ so $x = \theta + \frac{\pi}{3}$

$x_1 = \frac{\pi}{4} + \frac{\pi}{3}$ $\quad x_2 = \frac{5\pi}{4} + \frac{\pi}{3}$

$x_1 = \frac{3\pi}{12} + \frac{4\pi}{12}$ $\quad x_2 = \frac{15\pi}{12} + \frac{4\pi}{12}$

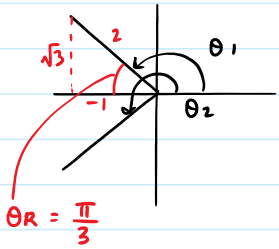
$x_1 = \frac{7\pi}{12}$ $\quad x_2 = \frac{19\pi}{12}$

c) $0 = 2\cos\left(x - \frac{\pi}{6}\right) + 1$

let $\theta = x - \frac{\pi}{6}$

$0 = 2\cos\theta + 1$

$-\frac{1}{2} = \cos\theta$



$\theta_1 = \frac{2\pi}{3}$

$\theta_2 = \frac{4\pi}{3}$

$\theta = x - \frac{\pi}{6}$ so $x = \theta + \frac{\pi}{6}$

$x_1 = \frac{2\pi}{3} + \frac{\pi}{6}$ $x_2 = \frac{4\pi}{3} + \frac{\pi}{6}$

$x_1 = \frac{4\pi}{6} + \frac{\pi}{6}$ $x_2 = \frac{8\pi}{6} + \frac{\pi}{6}$

$x_1 = \frac{5\pi}{6}$ $x_2 = \frac{9\pi}{6}$

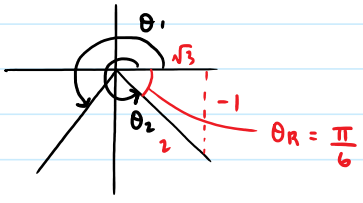
$x_2 = \frac{3\pi}{2}$

d) $4\sin 2x = -2$

let $\theta = 2x$

$4\sin\theta = -2$

$\sin\theta = -\frac{1}{2}$



$\theta_1 = \frac{7\pi}{6}$ $\theta_2 = \frac{11\pi}{6}$

per $= \frac{2\pi}{2} = \pi = \frac{12\pi}{12}$ $\theta = 2x$ so $x = \frac{\theta}{2}$

$0 \le x < 2\pi$ → $\frac{24\pi}{12}$ max

$x_1 = \frac{7\pi}{6} \cdot \frac{1}{2} = \frac{7\pi}{12}$ $x_2 = \frac{11\pi}{6} \cdot \frac{1}{2}$

$x_1 = \frac{7\pi}{12}$ $x_2 = \frac{11\pi}{12}$

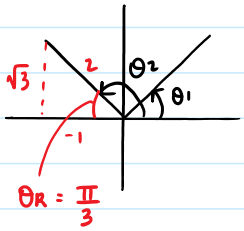
$x_3 = \frac{7\pi}{12} + \frac{12\pi}{12}$ $x_4 = \frac{11\pi}{12} + \frac{12\pi}{12}$

$x_3 = \frac{19\pi}{12}$ $x_4 = \frac{23\pi}{12}$

e) $\sin 3x = \frac{\sqrt{3}}{2}$

let $\theta = 3x$

$\sin\theta = \frac{\sqrt{3}}{2}$



$\theta_1 = \frac{\pi}{3}$ $\theta_2 = \frac{2\pi}{3}$

per $= \frac{2\pi}{3} = \frac{6\pi}{9}$ $\theta = 3x$ so $x = \frac{\theta}{3}$

up to $2\pi = \frac{18\pi}{9}$

$x_1 = \frac{\pi}{3} \cdot \frac{1}{3}$ $x_2 = \frac{2\pi}{3} \cdot \frac{1}{3}$

$x_1 = \frac{\pi}{9}$ $x_2 = \frac{2\pi}{9}$

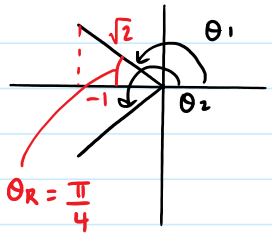
$x_3 = \frac{\pi}{9} + \frac{6\pi}{9}$ $\quad x_4 = \frac{2\pi}{9} + \frac{6\pi}{9}$ $\quad x_5 = \frac{7\pi}{9} + \frac{6\pi}{9}$ $\quad x_5 = \frac{8\pi}{9} + \frac{6\pi}{9}$

$x_3 = \frac{7\pi}{9}$ $\quad x_4 = \frac{8\pi}{9}$ $\quad x_5 = \frac{13\pi}{9}$ $\quad x_5 = \frac{14\pi}{9}$

f) $\sqrt{2}\cos\frac{1}{2}x = -1$

let $\theta = \frac{1}{2}x$

$\cos\theta = \frac{-1}{\sqrt{2}}$



$\theta_1 = \frac{3\pi}{4}$ $\quad \theta_2 = \frac{5\pi}{4}$

period $= \frac{2\pi}{1/2} = 4\pi$ $\quad \theta = \frac{1}{2}x$ so $x = 2\theta$

up to $2\pi = \frac{4\pi}{2}$

$x_1 = 2\left(\frac{3\pi}{4}\right)$ $\quad x_2 = 2\left(\frac{5\pi}{4}\right)$

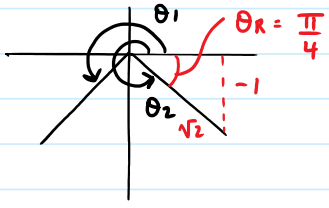
$x_1 = \frac{6\pi}{4}$ $\quad x_2 = \frac{10\pi}{4}$

$x_1 = \frac{3\pi}{2}$ $\quad x_2 = \frac{5\pi}{2}$ Too big!

2. a) $\sqrt{2}\sin 7x = -1$

let $\theta = 7x$

$\sin\theta = \frac{-1}{\sqrt{2}}$



$\theta_1 = \frac{5\pi}{4}$ $\quad \theta_2 = \frac{7\pi}{4}$

period $= \frac{2\pi}{7}$ $\quad \theta = 7x$ so $x = \frac{\theta}{7}$

$x_1 = \frac{5\pi}{4} \cdot \frac{1}{7}$ $\quad x_2 = \frac{7\pi}{4} \cdot \frac{1}{7}$

$x_1 = \frac{5\pi}{28}$ $\quad x_2 = \frac{7\pi}{28} = \frac{\pi}{4}$

general solution :

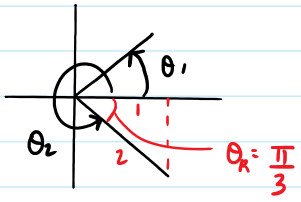
$x = \frac{5\pi}{28} + \frac{2\pi}{7}n, \quad n \in I$

$x = \frac{\pi}{4} + \frac{2\pi}{7}n, \quad n \in I$

b) $2\cos 3\left(x - \frac{\pi}{4}\right) = 1$

let $\theta = 3\left(x - \frac{\pi}{4}\right)$

$\cos\theta = \frac{1}{2}$



$\theta_1 = \frac{\pi}{3}$ $\theta_2 = \frac{5\pi}{3}$

period $= \frac{2\pi}{3}$

$\theta = 3\left(x - \frac{\pi}{4}\right)$ so $x = \frac{\theta}{3} + \frac{\pi}{4}$

$x_1 = \frac{\pi}{3}\cdot\frac{1}{3} + \frac{\pi}{4}$ $x_2 = \frac{5\pi}{3}\cdot\frac{1}{3} + \frac{\pi}{4}$

$x_1 = \frac{\pi}{9} + \frac{\pi}{4}$ $x_2 = \frac{5\pi}{9} + \frac{\pi}{4}$

$x_1 = \frac{4\pi}{36} + \frac{9\pi}{36}$ $x_2 = \frac{20\pi}{36} + \frac{9\pi}{36}$

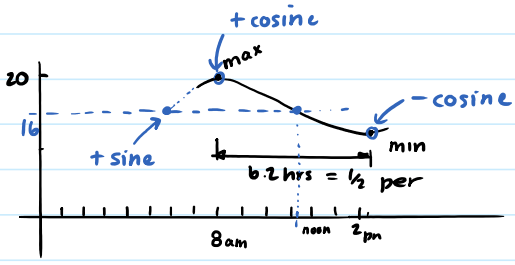
$x_1 = \frac{13\pi}{36}$ $x_2 = \frac{29\pi}{36}$

general solution :

$x = \frac{13\pi}{36} + \frac{2\pi}{3}n,\ n \in I$

$x = \frac{29\pi}{36} + \frac{2\pi}{3}n,\ n \in I$

3.



per = 2(6.2) = 12.4

$B = \frac{2\pi}{12.4}$ or $\frac{\pi}{6.2}$

$A = \frac{|20-12|}{2} = 4$ $D = 20 - 4 = 16$

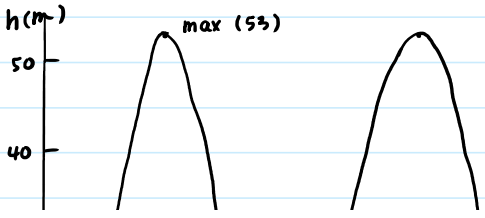
$y = 4\cos\frac{2\pi}{12.4}(x-8) + 16$ $y = 4\sin\frac{2\pi}{12.4}(x-4.9) + 16$

$y = -4\cos\frac{2\pi}{12.4}(x-14.2) + 16$

4. a) $A = 26$

$B = \frac{2\pi}{50}$ so per = 50

$C = 25$



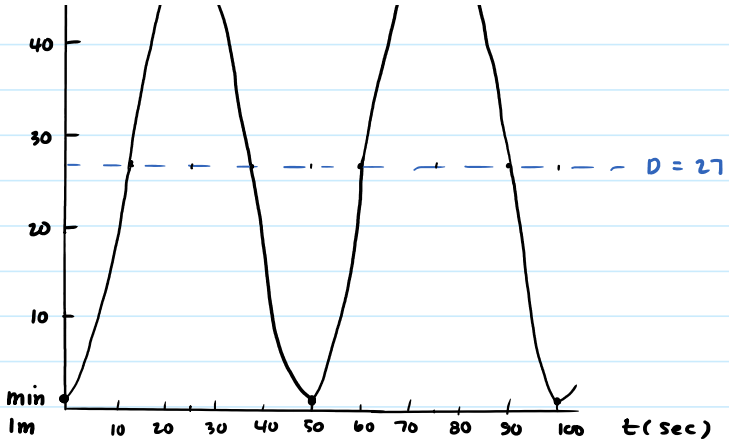
50

C = 25

D = 27

min = 27 − 26 = 1 m

max = 27 + 26 = 53 m



b) max height reached after 25 seconds

c) max height = 53m

d) it will take 50 seconds (1 full period)

e) Ferris wheel is 1m above the ground

5. r = 25m

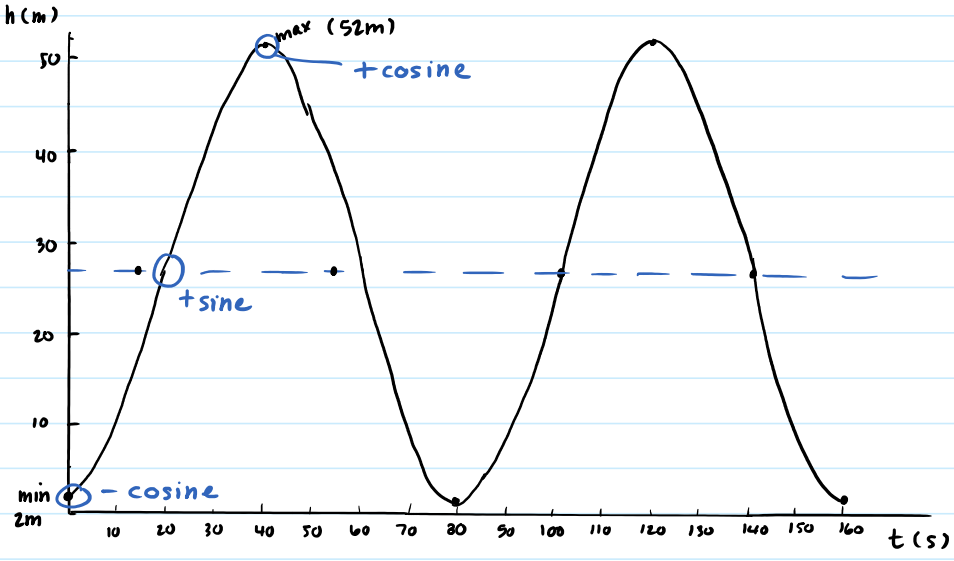
per = 80 sec

min = 2 m

A = 25

D = 2+25 = 27

$\beta = \frac{2\pi}{80}$ or $\frac{\pi}{40}$



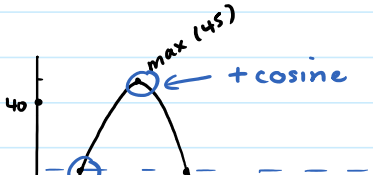
$y = 25 \cos \frac{2\pi}{80}(x - 40) + 27$

$y = 25 \sin \frac{2\pi}{80}(x - 20) + 27$

$y = -25 \cos \frac{2\pi}{80} x + 27$

b. r = 20 cm

centre at 25 cm

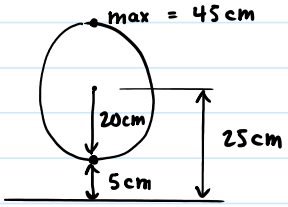


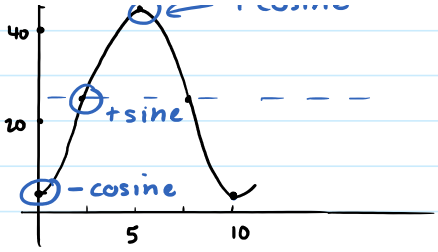
b. r = 20 cm

centre at 25 cm

period : 10 sec

min height at t = 0 s





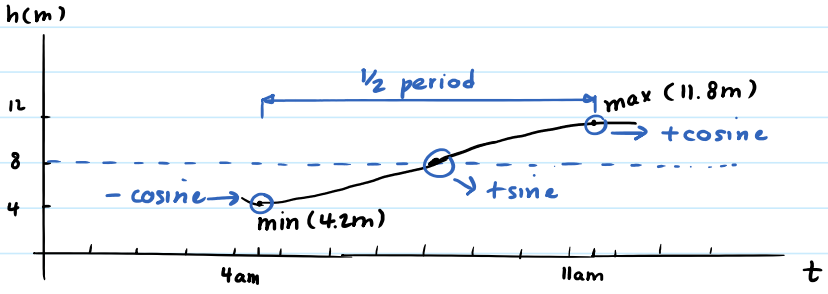
$A = 20$ $\quad B = \frac{2\pi}{10}$ or $\frac{\pi}{5}$

$D = 25$

$y = 20\cos\frac{2\pi}{10}(x-5) + 25$ $\qquad y = 20\sin\frac{2\pi}{10}(x-2.5) + 25$

$y = -20\cos\frac{2\pi}{10}x + 25$

7. a)



½ per = 7 hrs $\quad$ so $\quad$ 1 per = 14 hrs

$Amp = \frac{|11.8 - 4.2|}{2} = 3.8$ $\qquad B = \frac{2\pi}{14}$ or $\frac{\pi}{7}$

$D = 11.8 - 3.8 = 8.0$

$y = 3.8\cos\frac{2\pi}{14}(x - 11.5) + 8.0$ $\qquad y = 3.8\sin\frac{2\pi}{14}(x - 8.0) + 8.0$

$y = -3.8\cos\frac{2\pi}{14}(x - 4.5) + 8.0$

b) height at 1:15 pm ? $\qquad x = 13.25$

$y = 3.8\cos\frac{2\pi}{14}(13.25 - 11.5) + 8.0$

$$y = 3.8 \cos \frac{2\pi}{14} (13.25 - 11.5) + 8.0$$

$$y = 3.8 \cos 0.7854 + 8.0$$

$$y = 2.6870 + 8.0$$

$y = 10.7$ m

c) 1st high tide @ 11:30 am → next one ?

11:30 am + 14 hrs (1 period) → 1:30 am next day