Pre-Calculus 12 Practice Exam A

vertical displacement

Multiple-Choice: Part 1 Calculator Not Permitted

phase shift right 2

1. Consider the graph of $y = -3\cos\frac{\pi(x-2)}{10} + 4$. Which statement is $\boxed{\text{false?}}$

I.	The amplitude is 3 T
II.	The period is 10 F
III.	The phase shift is 2 to the right T
IV.	The vertical displacement is 4 up T

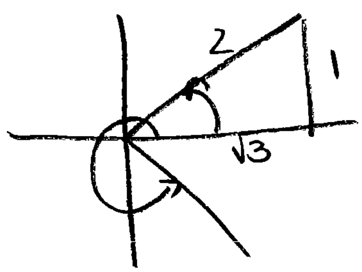
~~A. I~~
B. II
~~C. III~~
D. IV

Amp $= |-3| = 3$

period $= \frac{2\pi}{\pi/10}$
$= 2\pi \cdot \frac{10}{\pi}$
$= 20$

phase shift
(x−2) means
right 2

2. Solve: $\cos x = \frac{\sqrt{3}}{2}$, $0 \le x < 2\pi$



A. $\frac{\pi}{6}, \frac{5\pi}{6}$

B. $\frac{\pi}{6}, \frac{11\pi}{6}$

C. $\frac{\pi}{3}, \frac{2\pi}{3}$

D. $\frac{\pi}{3}, \frac{5\pi}{3}$

ref $\angle = \frac{\pi}{6}$

Quad 1 $x = \pi/6$

Quad 2 $x = 2\pi - \frac{\pi}{6}$

$x = \frac{12\pi}{6} - \frac{\pi}{6}$

$x = \frac{11\pi}{6}$

3. Determine an expression for all angles coterminal with a standard position angle measuring 120°. Express your answer in radians.

A. $\frac{5\pi}{6}+\pi n$, n is an integer

B. $\frac{2\pi}{3}+\pi n$, n is an integer

C. $\frac{5\pi}{6}+2\pi n$, n is an integer

D. $\frac{2\pi}{3}+2\pi n$, n is an integer

Change to Radians

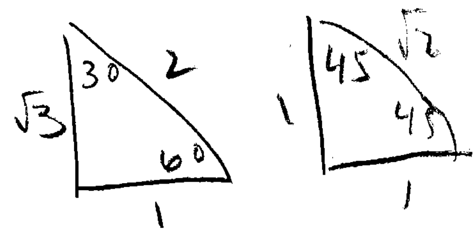
$120^\circ\left(\frac{\pi}{180^\circ}\right)$

$= \frac{12\pi}{18}$

$= \frac{2\pi}{3}$

coterminal $+2\pi n$

$\frac{2\pi}{3}+2\pi n$



4. Determine the exact value of $\tan 75^\circ$.

A. $2+\sqrt{3}$

B. $-2-\sqrt{3}$

C. $\frac{5+\sqrt{3}}{4}$

D. $\frac{3+\sqrt{3}}{\sqrt{3}}$

$\tan(30^\circ+45^\circ)$

$\frac{\tan 30+\tan 45}{1-(\tan 30)(\tan 45)}$

$=\frac{\frac{1}{\sqrt{3}}+1}{1-\left(\frac{1}{\sqrt{3}}\right)(1)}$

$=\frac{\left(\frac{1}{\sqrt{3}}+1\right)\sqrt{3}}{\left(1-\frac{1}{\sqrt{3}}\right)\sqrt{3}}$

$=\frac{1+\sqrt{3}}{\sqrt{3}-1}$

$=\frac{1+\sqrt{3}}{\sqrt{3}-1}\cdot\frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$

$=\frac{\sqrt{3}+1+\sqrt{3}+1}{3+\sqrt{3}-\sqrt{3}-1}$

$=\frac{4+2\sqrt{3}}{2}=2+\sqrt{3}$

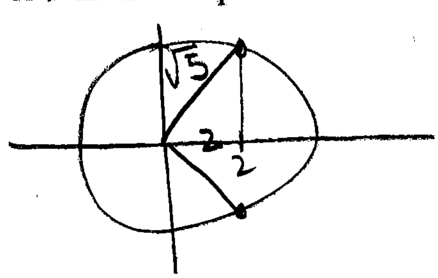
5. A point with an x value of 2 lies on the circle with equation $x^2+y^2=5$. This point also lies on the terminal arm of θ in standard position. Determine the value of $\sec\theta$.

A. $\frac{\sqrt{5}}{2}$

B. $\frac{5}{2}$

C. $\frac{2}{\sqrt{5}}$

D. $\frac{2}{5}$



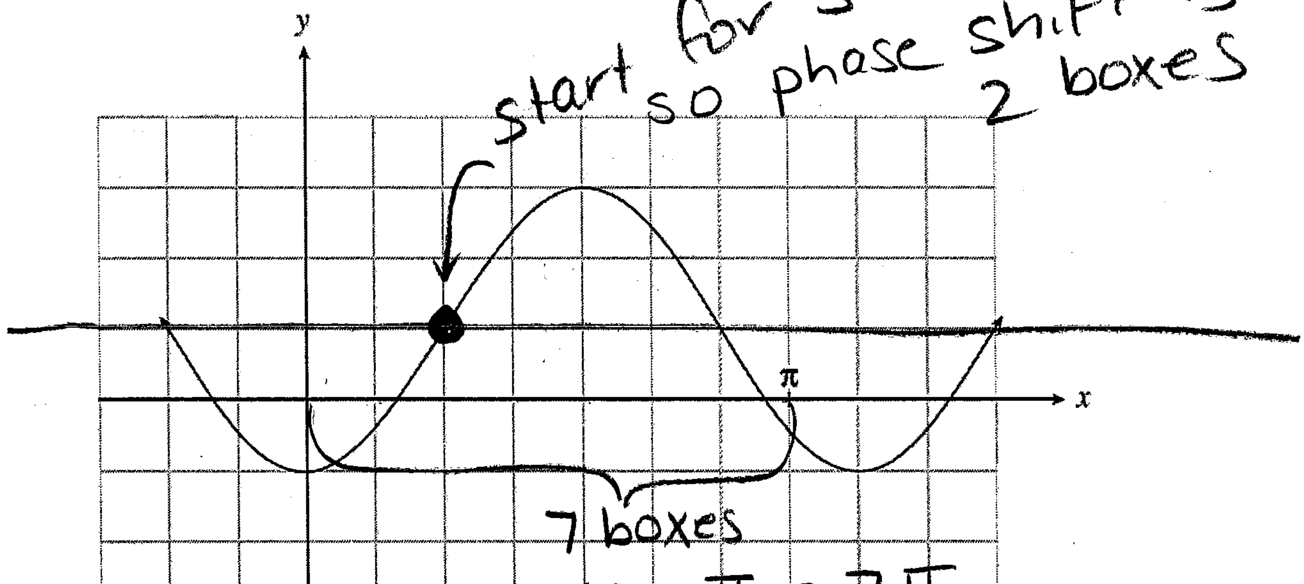
radius $=\sqrt{5}$

since $\cos\theta=\frac{x}{r}$ $\quad\sec\theta=\frac{r}{x}$

$\sec\theta=\frac{\sqrt{5}}{2}$

6. The graph of $y = 2\sin b(x-c)+1$ is shown below. Determine a value of c.

start for sin so phase shift is 2 boxes



so $\pi = \frac{7\pi}{7}$

each box $= \frac{\pi}{7}$

A. $-\frac{2\pi}{2}$

B. 2

C. $\frac{\pi}{4}$

D. $\frac{2\pi}{7}$

7. Determine all restrictions for the expression $\frac{\tan x}{\cos x - 1}$.

A. $\cos x \neq 0$

B. $\cos x \neq 1$

C. $\sin x \neq 0$, $\cos x \neq 1$

D. $\cos x \neq 0$, $\cos x \neq 1$

tan x has asymptotes

$\tan x = \frac{\sin x}{\cos x}$

when $\cos x = 0$ so $\cos x \neq 0$

$\cos x - 1 \neq 0$

$\cos x \neq 1$

8. Solve: $\sin x = -\cos x$ ($-\pi \le x \le \pi$

$\sin x = -\cos x$

$\frac{\sin x}{\cos x} = -1$

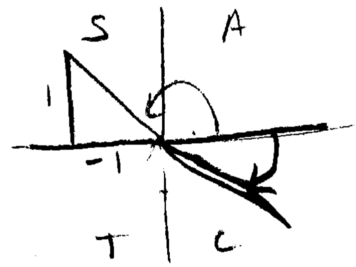
$\tan x = -1$

A. $-\frac{\pi}{4}, \frac{3\pi}{4}$

B. $\frac{\pi}{4}, \frac{3\pi}{4}$

C. $\frac{3\pi}{4}, \frac{7\pi}{4}$

D. $\frac{3\pi}{4}, \frac{5\pi}{4}$



ref $\angle = \frac{\pi}{4}$

Quad 2 $\pi - \frac{\pi}{4} = \frac{3\pi}{4}$

Quad 4 $-\pi/4$

9. Simplify: $\frac{\csc\theta - \sin\theta}{\sec\theta - \cos\theta}$

A. $\cot^2\theta$

B. $\cot^3\theta$

C. $\tan^2\theta$

D. $\tan^3\theta$

$$\frac{\frac{1}{\sin\theta} - \sin\theta}{\frac{1}{\cos\theta} - \cos\theta}$$

$$= \frac{\frac{1-\sin^2\theta}{\sin\theta}}{\frac{1-\cos^2\theta}{\cos\theta}}$$

$$= \frac{\frac{\cos^2\theta}{\sin\theta}}{\frac{\sin^2\theta}{\cos\theta}}$$

$$= \frac{\cos^2\theta}{\sin\theta} \cdot \frac{\cos\theta}{\sin^2\theta}$$

$$= \frac{\cos^3\theta}{\sin^3\theta}$$

$$= \cot^3\theta$$

10. Solve: $2^{3x-1} = 8^{2x+1}$

A. $x = -\frac{4}{3}$

B. $x = -1$

C. $x = -\frac{2}{3}$

D. $x = -\frac{3}{4}$

$$2^{3x-1} = (2^3)^{2x+1}$$

$$2^{3x-1} = 2^{6x+3}$$

$$3x - 1 = 6x + 3$$

$$-4 = 3x$$

$$-\frac{4}{3} = x$$

11. Express $\log \frac{x^2}{10y^3}$ in terms of $\log x$ and $\log y$.

A. $2\log x - 1 - 3\log y$
B. $2\log x - 1 + 3\log y$
C. $2\log x - 10 - 3\log y$
D. $2\log x - 10 + 3\log y$

$\log x^2 - \log 10y^3$
$\log x^2 - [\log 10 + \log y^3]$
$\log x^2 - \log 10 - \log y^3$
$2\log x - 1 - 3\log y$

12. Evaluate: $\log_3 \sqrt{27}$

A. $\frac{2}{9}$

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. $\frac{9}{2}$

$\log_3 (27)^{\frac{1}{2}}$
$\log_3 (3^3)^{1/2}$
$\log_3 3^{3/2}$
$\frac{3}{2} \log_3 3$
$\frac{3}{2}(1)$
$3/2$

13. Bart and Arnie presented separate solutions to the statement:

"Write $\log_2 x + \log_4 y$ as a single log."

Bart	Arnie
$\log_2 x + \log_4 y$	$\log_2 x + \log_4 y$
$= \dfrac{\log_4 x}{\log_4 2} + \log_4 y$	$= \log_2 x + \dfrac{\log_2 y}{\log_2 4}$
$\dfrac{\log_4 x}{1/2} + \log_4 y$	$\dfrac{\log_2 y}{2}$
$= 2\log_4 x + \log_4 y$	$= \log_2 x + \frac{1}{2}\log_2 y$
$= \log_4 x^2 y$	$= \log_2 x\sqrt{y}$

used change of base

$\log_c x = \dfrac{\log_b x}{\log_b c}$

$\log_4 2$
$= \log_4 4^{1/2} = \frac{1}{2}$

change of base

$\log_2 2^2$
$= 2\log_2 2$
2

Which statement is true?

A. Only Bart is correct.
B. Only Arnie is correct.
C. They are both wrong.
D. They are both correct.

$\log_4 x^2 y = \log_2 x\sqrt{y}$

$= \dfrac{\log_2 x^2 y}{\log_2 4}$

$= \dfrac{\log_2 x^2 y}{2}$

$\frac{1}{2}\log_2 x^2 y$

$\log_2 (x^2)^{1/2} y^{1/2}$

$\log_2 x\sqrt{y}$

14. Which statement must be true for $f(x) = \log_{\frac{1}{2}} x$ when $x_2 > x_1$?

A. $f(x_1) > f(x_2)$
B. $f(x_2) > f(x_1)$
C. $f(x_1) > 0, \quad f(x_2) < 0$
D. $f(x_2) > 0, \quad f(x_1) < 0$

pick #'s for x_2 and x_1 (ones that can be written in base $\frac{1}{2}$)

$x_2 > x_1$

$\frac{1}{2} > \frac{1}{8}$

$\log_{\frac{1}{2}} x_1$ $\quad$ $\log_{\frac{1}{2}} x_2$

$\log_{\frac{1}{2}} \frac{1}{8}$ $\quad$ $\log_{\frac{1}{2}} \frac{1}{2}$

$\log_{\frac{1}{2}} (\frac{1}{2})^3$ $\quad$ 1

$3 > 1$

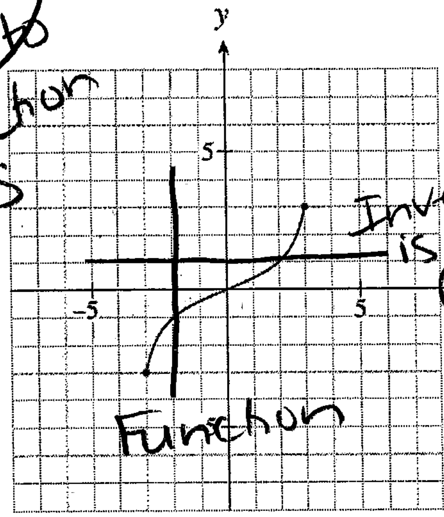
Switch x-and-y

If the inverse is a function the original function passes the Horizontal line test

15. For which graph is the relation and its inverse both functions?

For the original to be a function must pass vertical line test

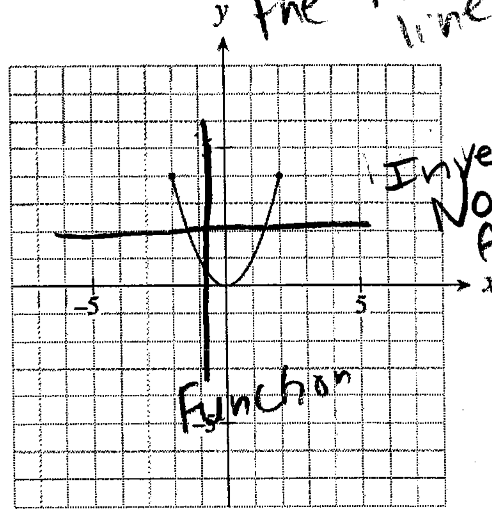
A.



Inverse is a function

Function

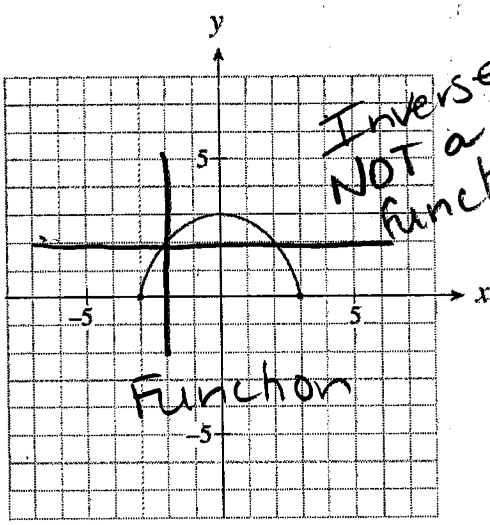
B.



Inverse Not a funct-

Function

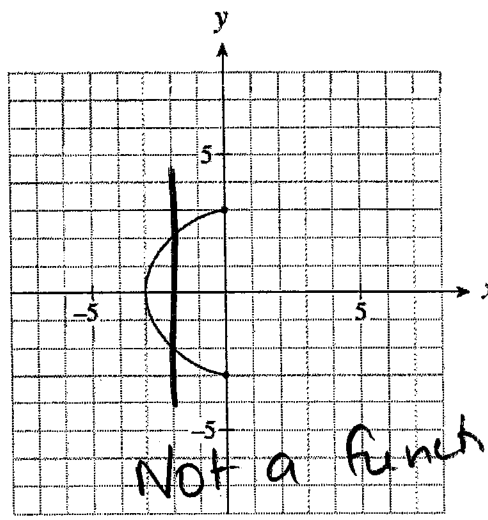
C.



Inverse NOT a function

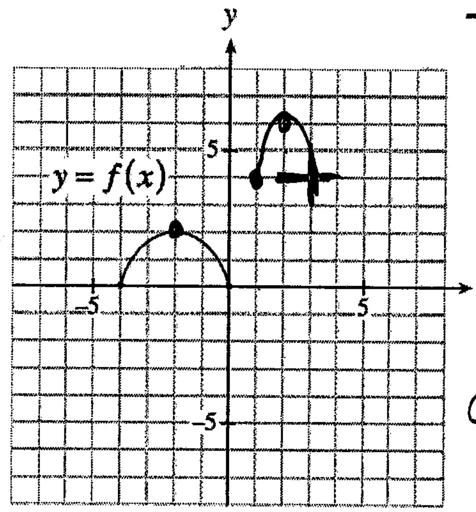
Function

D.



Not a function

16. The graph of $y = f(x)$ is shown below.



x	y
-4	0
-2	2
0	0

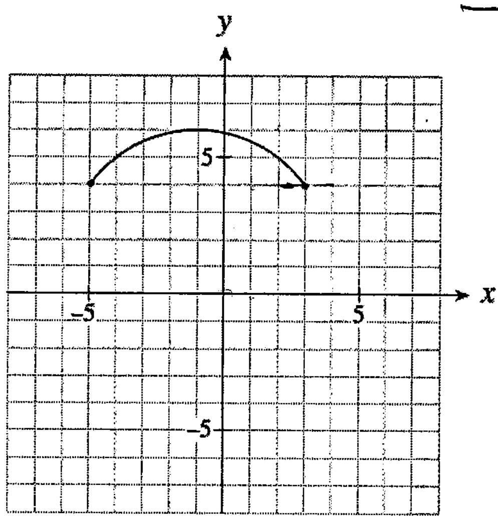
b = 2
divide x's by 2

x	y
-2	0
-1	2
0	0

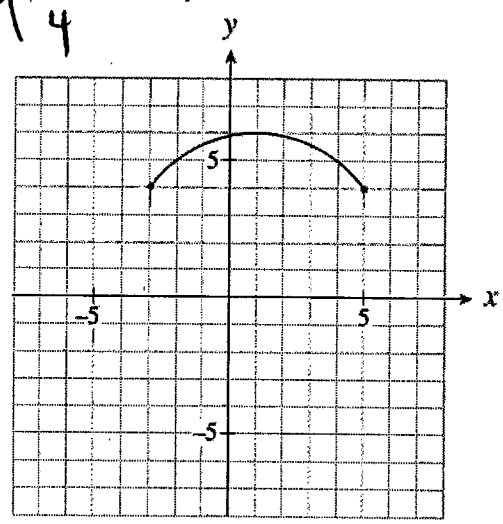
Which graph represents the graph of $y = f(2(x-3))+4$?

$\xrightarrow{}$ 3 ↑ 4

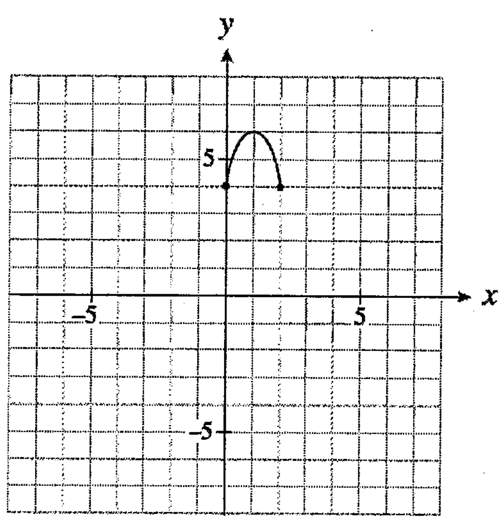
A.



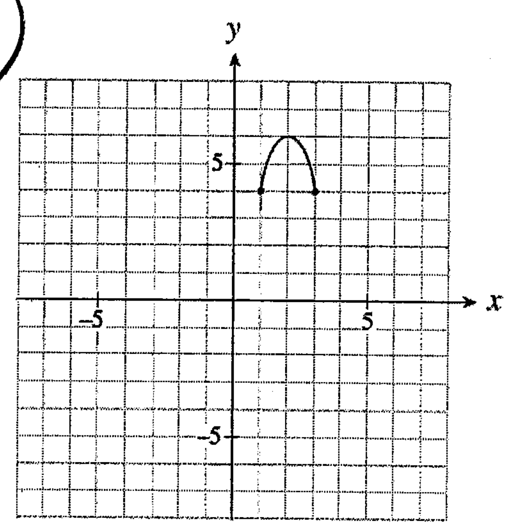
B.



C.



D.



17. Consider the following transformations on the graph of $y = f(x)$.

I.	$y = f(x+2)$	← subtract 2 from x
II.	$y = 2f(x)$	← mult y by 2
III.	$y = f(-x)$	← divide x by (-1)
IV.	$y = -f(x)$	← Mult y by (-1)

Which transformations will have no effect on the zeros of the original graph of $y = f(x)$?

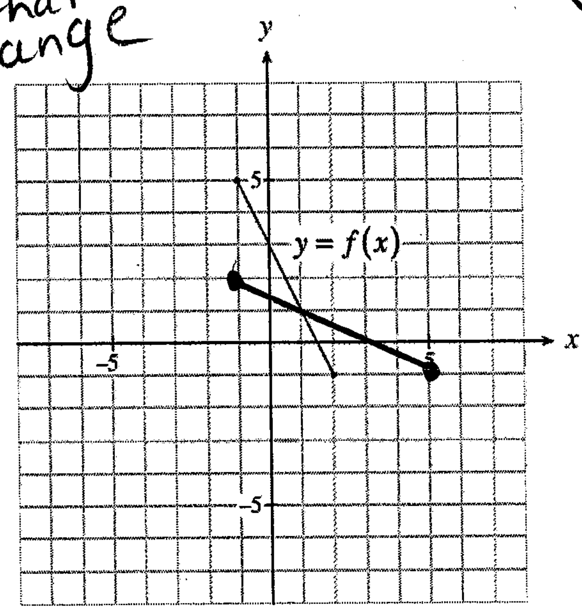
x-intercepts (x, 0)

A. I and II only
B. ~~II and III only~~
C. II and IV only
D. III and IV only

18. The graph of $y = f(x)$ as shown below is transformed to $x = f(y)$. Determine all invariant points.

inverse

points that don't change



A. (0, 3)
B. (1, 1)
C. (2, −1)
D. (1, 1) and (2, −1)

19. The point $P(4,\ 6)$ lies on the graph of $y = f(x)$. Which point must lie on the graph of $y = -\frac{1}{2}f\left(\frac{1}{2}x+2\right)$?

$y = -\frac{1}{2}f\left(\frac{1}{2}(x+4)\right)$

A. $(7, -3)$

B. $(4, -3)$

~~C. $(1, -3)$~~

D. $(-2, -3)$

$4 \mid 6$ $\quad a = -\frac{1}{2}$ $\quad b = \frac{1}{2}$ $\quad 8 \mid -3$ $\quad \leftarrow 4$ left 4 $\quad 4 \mid -3$

mult y's by $-\frac{1}{2}$

divide x by $\frac{1}{2}$

20. Which of the following functions are polynomial functions?

I.	$y = x^3 - \sqrt{2}x^2 + x + 3$ ✓
II.	$y = x^3 - \frac{2}{x^2} - x + 3$ X
III.	$y = x^3 - 2x^{1.5} + x + 3$ X
IV.	$y = x^3 - \frac{1}{2}x^2 - x + 3$ ✓

A. III only

~~B. IV only~~

C. I and IV only

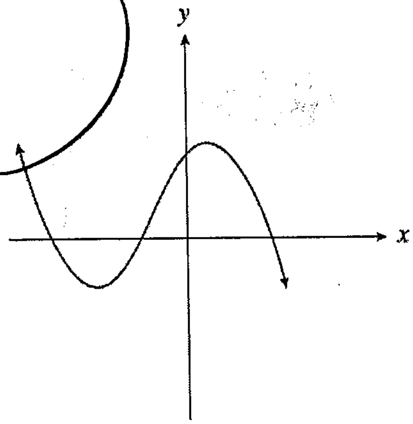
~~D. II and III only~~

Since $a<0$
end behaviour
up into Quad 2
down into Quad 4

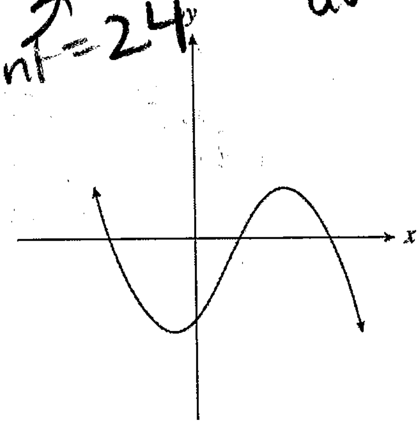
21. Which sketch best represents the graph of $y = ax^3 - bx^2 + cx + 24$ if $a < 0$?

y-int = 24

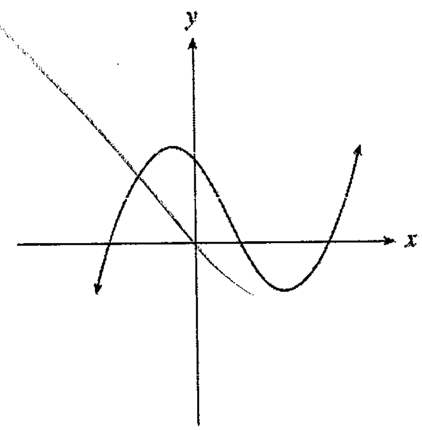
A.



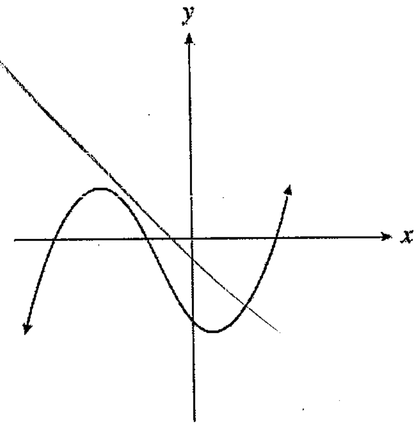
B.



C.



D.



22. Which three expressions are factors of $9x^3 - 36x^2 - 4x + 16$? $= f(x)$

$f(4) = 9(4)^3 - 36(4)^2 - 4(4) + 16$

$f(4) = 576 - 576 - 16 + 16 = 0$

$$\begin{array}{r|rrrr} -4 & 9 & -36 & -4 & 16 \\ & & -36 & 0 & 16 \\ \hline & 9 & 0 & -4 & 0 \end{array}$$

I.	$x-4$
II.	$x+4$
III.	$3x-2$
IV.	$3x+2$

$f(x) = (x-4)(9x^2-4)$

$f(x) = (x-4)(3x-2)(3x+2)$

A. I, II, III only
B. ~~I, II, IV only~~
C. I, III, IV only
D. II, III, IV only

$f(x)$

23. When $x^3 - 2kx^2 + 3k^2x - 15$ is divided by $x - 2$, the remainder is 1. Determine all values for k.

$f(2) = R$

$f(2) = 1$

A. $k = -4$

B. $k = \frac{17}{8}$

C. $k = -\frac{2}{3}, 2$

D. $k = \frac{2}{3}, -2$

$2^3 - 2k(2)^2 + 3k^2(2) - 15 = 1$

$8 - 8k + 6k^2 - 15 = 1$

$6k^2 - 8k - 8 = 0$

$3k^2 - 4k - 4 = 0$

$3k^2 - 6k + 2k - 4 = 0$

$3k(k-2)\ 2(k-2) = 0$

$(k-2)(3k+2) = 0$

$\times = -12$

$-6, 2 \quad + = -4$

24. Given $f(x) = x + 2$ and $g(x) = x^2 + 3x - 1$, determine the value of $f(g(3))$.

A. 16

~~B. 17~~

C. 19

~~D. 39~~

$g(3) = 3^2 + 3(3) - 1$

$g(3) = 17$

$f(g(3)) = f(17)$

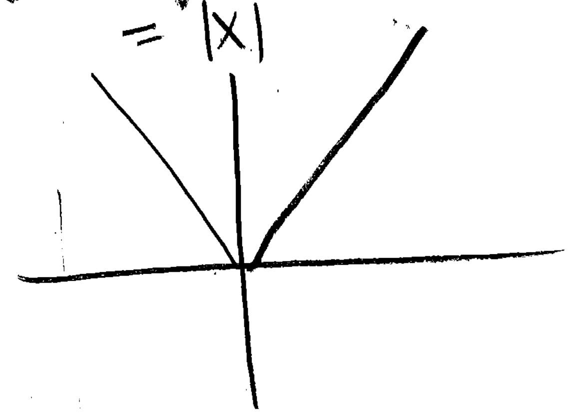
$= 17 + 2$

25. Consider the graphs of the functions $f(x) = x^2$ and $g(x) = \sqrt{f(x)}$. Which row describes the domains and range of $g(x)$?

	Domain	Range
A.	all reals ✓	all reals
B.	has restrictions	has restrictions ✓
C.	has restrictions	all reals
D.	all reals ✓	has restrictions ✓

$g(x) = \sqrt{x^2}$

$= |x|$



x	$g(x)$
-2	$\sqrt{(-2)^2} = \sqrt{4} = 2$
-1	$\sqrt{(-1)^2} = \sqrt{1} = 1$
0	0
1	1
2	2

26. Determine the range of the function $y = \sqrt{3x-9} + 2$.

$y = \sqrt{3(x-3)} + 2$

~~A. $y \geq 0$~~

B. $y \geq 2$

C. $y \geq 3$

D. $y \geq 9$

$y = \sqrt{x}$ range $y \geq 0$

$y = \sqrt{x} + 2$ range $y \geq 2$

Multiple-Choice: Part 2 Calculator Permitted

27. Determine the smallest zero for $y = 4\sin 3\theta + 2$ in the interval $2\pi \leq \theta \leq 3\pi$.

These are both smaller than π

A. 0.38

B. 1.22

~~C. 6.66~~

D. 7.50

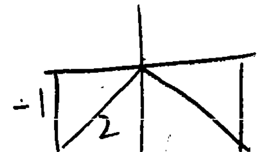
let $3\theta = x$

$0 = 4\sin x + 2$

$-2 = 4\sin x$

$-\frac{2}{4} = \sin x$

$-\frac{1}{2} = \sin x$



ref $\angle = \frac{\pi}{6}$

$x = \frac{7\pi}{6}$ $x = \frac{11\pi}{6}$

$\theta = 7\pi/18$ $\theta = \frac{11\pi}{18}$

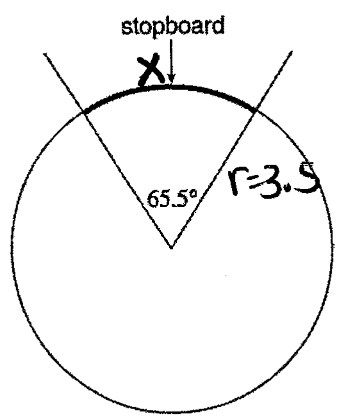
$\frac{36\pi}{18} \leq \theta \leq \frac{54\pi}{18}$

period $= \frac{2\pi}{3} = \frac{12\pi}{18}$

solution $= \frac{7\pi}{18} +$ period

Keep adding on period until you get a solution

28. In high school, a shot put is thrown out of a circle with a radius of 3.5 feet. A curved wooden "stopboard" is placed in an arc around part of this circle. The central angle is 65.5°. Determine the length of the curved "stopboard."



$$\frac{x}{2\pi(3.5)} = \frac{65.5}{360}$$

$$x = \frac{65.5(2\pi)(3.5)}{360}$$

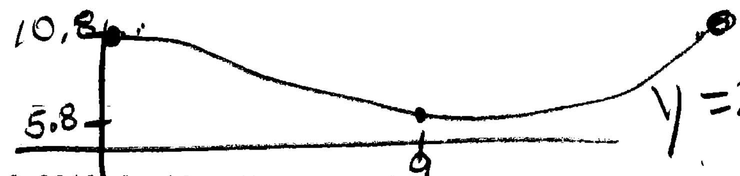
$$x = 4.00117$$

~~A. 3.5 feet~~

B. 4.0 feet

~~C. 4.5 feet~~

D. 5.0 feet



$y = 2.5\cos\frac{2\pi}{18}x + 8.3$

29. On Oct. 2, 2010, the tide at New Westminster reached a maximum height of 10.8 feet at midnight. At 9 am the tide reached the next minimum height of 5.8 feet. Assuming the relationship is sinusoidal, what was the height of the tide at 7 am?

Sub $x=7$ into the equation →

A. 6.1 feet
B. 6.4 feet
C. 8.7 feet
D. 9.9 feet

$Amp = \frac{|10.8-5.8|}{2}$

$Amp = \frac{5}{2} = 2.5$

No phase shift if cos

$period = 2(9-0) = 18$

$B = \frac{2\pi}{18}$

$VD = 10.8 - 2.5$

$VD = 8.3$

30. Solve: $7 = 2^{x+1}$

A. –0.64
B. 1.36
C. 1.81
D. 3.81

$\log 7 = \log 2^{x+1}$

$\log 7 = (x+1)\log 2$

$\log 7 = x\log 2 + \log 2$

$\log 7 - \log 2 = x\log 2$

$\frac{\log 7 - \log 2}{\log 2} = x$

31. The population in a particular community is increasing at an annual rate of 6.5%. Assume this trend will continue. In how many years will the present population of 12 000 grow to 32 000?

A. 15.5
B. 15.6
C. 15.8
D. 16.1

$A = A_0(c)^{t/T}$

$\frac{32000}{12000} = \frac{12000(1.065)^{t/1}}{12000}$

$A = 32000$

$A = 12000$

$c = 100\% + 6.5\%$

$c = 106.5\% = 1.065$

$t =$

$T = 1$

$\frac{32}{12} = 1.065^t$

$\log\frac{8}{3} = \log 1.065^t$

$\log\frac{8}{3} = t\log 1.065$

$\frac{\log 8/3}{\log 1.065} = t$

32. In a study which compared the pH of urine and tears, the following data was collected.

Urine

$6 = -\log[H^+]$

$-6 = \log[H^+]$

$10^{-6} = [H^+]$

	Urine	Tears
Joe	6.2	7.6
Bob	6.3	7.4
Bill	5.5	7.5
Average	**6.0**	**7.5**

Tears

$7.5 = -\log[H^+]$

$-7.5 = \log[H^+]$

$10^{-7.5} = [H^+]$

On average, how many times more alkaline are tears than urine?

$\frac{10^{-6}}{10^{-7.5}} = 10^{-6-(-7.5)}$

$= 10^{1.5}$

$= 31.6$

A. 1.3
B. 1.5
C. ~~15.0~~
D. 31.6

33. When a polynomial $P(x)$ is divided by $x+3$, the remainder is 2. Which point must be on the graph of the corresponding function $y = P(x)$.

A. $(-3, -2)$
B. ~~$(-3, 0)$~~
C. $(-3, 2)$
D. ~~$(3, 2)$~~

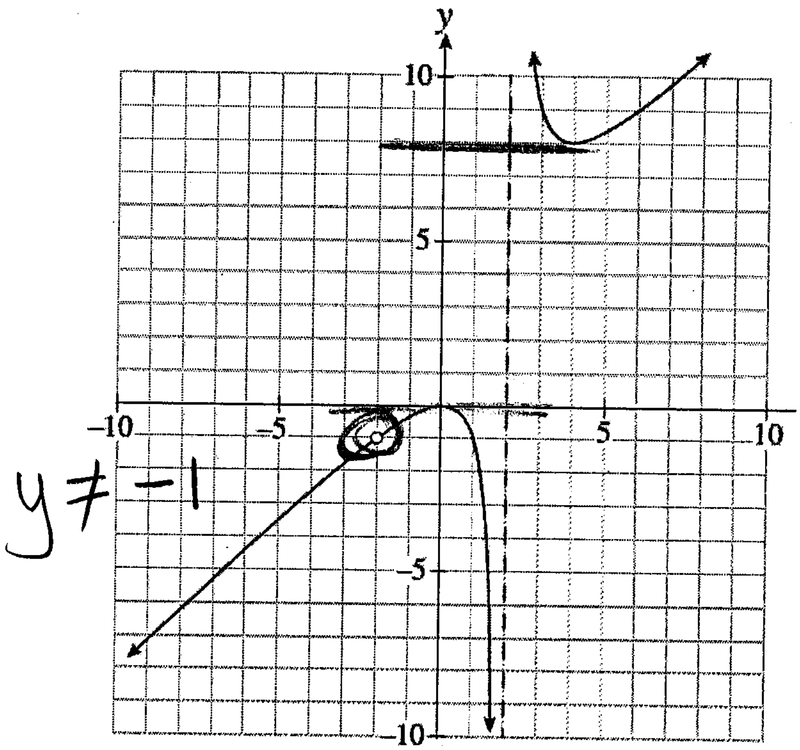
Remainder = P(a) if the polynomial is being divided by (x-a)

so $P(-3) = 2$

x-value

y-value

34. Determine the range of the rational function graphed below.



A $\{y:\ y \in \mathbb{R}\}$

B $\{y: y \leq 0,\ y \geq 8\ y \in \mathbb{R}\}$

C $\{y: y \neq 2\ y \in \mathbb{R}\}$

D $\{y:\ y \leq 0,\ y \neq -1,\ y \geq 8\ \ y \in \mathbb{R}\}$

35. The graph of $y = f(x)$ is stretched horizontally by a factor of $\frac{1}{4}$. Determine the equation of the transformed graph.

Horizontally stretched factor $\frac{1}{4}$ means $b = 4$

A. $y = \frac{1}{4}f(x)$

B. $y = 4f(x)$

C. $y = f\left(\frac{1}{4}x\right)$

D. $y = f(4x)$

36. How many terms are there in the series defined by $\sum_{k=4}^{31} 2(3)^{k-1}$

~~A. 27~~
B. 28
C. 30
D. 31

$n = 31 - 4 + 1$

$n =$

37. Determine the sum of the first 10 terms of the geometric series defined by $\frac{2}{3} - 2 + 6 - 18 + \ldots$

A. -9 841.33
~~B. 3 280.67~~
C. 9 841.67
D. 19 682.67

$t_1 = \frac{2}{3}$

$r = \frac{-2}{2/3} = -2\left(\frac{3}{2}\right) = -3$

$n = 10$

$S_{10} = \frac{\frac{2}{3}\left((-3)^{10} - 1\right)}{-3-1}$

38. In a geometric sequence, $t_2 = 480$ and $t_7 = -15$. Determine the common ratio, r.

A. -3

B. -2

C. $-\frac{1}{3}$

D. $-\frac{1}{2}$

$480 = t_1(r)^{2-1}$ $\quad -15 = t_1(r)^{7-1}$

$480 = t_1(r)$ $\quad -15 = t_1(r)^6$

$\frac{-15}{480} = \frac{t_1 r^6}{t_1 r}$

$-\frac{1}{32} = r^5$

$\sqrt[5]{\frac{-1}{32}} = r \quad r = -\frac{1}{2}$

39. The sum of the infinite geometric series $t + t^2 + t^3 + t^4 + \cdots$ is $4t, t \neq 0$. The value of t is

A. $\frac{4}{3}$

B. $\frac{3}{4}$

C. $\frac{1}{2}$

D. $\frac{1}{4}$

$S = 4t$

$t_1 = t$

$r = \frac{t^2}{t} = t$

$S = \frac{t_1}{1-r}$

$4t = \frac{t}{1-t}$

$(1-t)4t = t$

$4t - 4t^2 = t$

$0 = 4t^2 - 3t$

$0 = t(4t - 3)$

~~$t = 0$~~

$t = \frac{3}{4}$

40. Solve $\ln(-x) + \ln 6 = 2$

A. -10.8

B. -1.2315

C. 17.2411

D. 55

$$\ln(-x \cdot 6) = 2$$
$$\ln(-6x) = 2$$
$$e^{2} = -6x$$
$$\frac{e^{2}}{-6} = x$$
$$x = -1.2315$$

41. Identify the domain of the function $y = \ln(4x + 16) - 2$

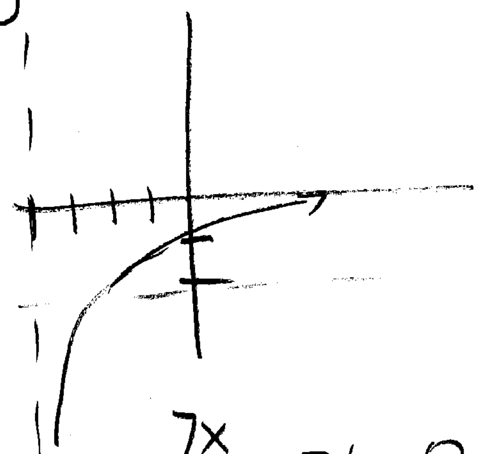
A. $x > -4$

B. $x < -2$

C. $x > 4$

D. $x < 4$

$$y = \ln(4(x+4)) - 2$$



42. Solve $e^{7x} + 2 = 38.8$

A. 0.1922

B. 0.2237

C. 0.5151

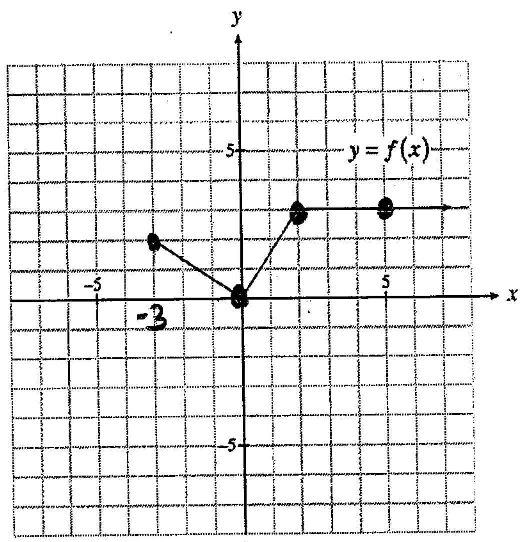
D. 0.2107

$$e^{7x} = 36.8$$
$$\ln e^{7x} = \ln 36.8$$
$$7x \ln e = \ln 36.8$$
$$7x = \ln 36.8$$
$$x = \frac{\ln 36.8}{7}$$
$$x = 0.51507$$

Written Response

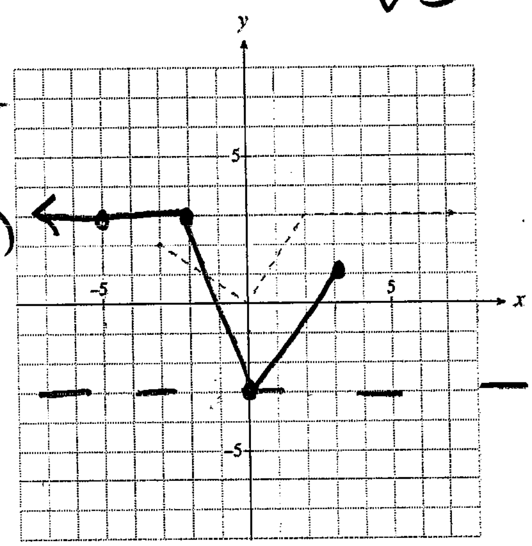
1. The graph of $y = f(x)$ is shown. On the grid provided, sketch the graph of $y = 2f(-x) - 3$.

$a = 2 \quad b = -1 \quad \downarrow 3$



Mult y's by 2
divide x's by (-1)

3	4
0	0
-2	6
-5	6



-3	2
0	0
2	3
5	3

2. Solve algebraically: $\log_{15}(3 - x) + \log_{15}(1 - x) = 1$

$\log_{15}[(3-x)(1-x)] = \log_{15} 15$

$(3-x)(1-x) = 15$

$3 - 4x + x^2 = 15$

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$\cancel{x = 6} \quad \boxed{x = -2}$

$3 - x > 0$
$-x > -3$
$x < 3$

$1 - x > 0$
$-x > -1$
$x < 1$

3. Solve algebraically $0 \le \theta < 2\pi$. Give exact values if possible, otherwise round answers to two decimal places.

$$3sin^2\theta + 5cos\theta = 1$$

$3\sin^2\theta + 5\cos\theta = 1$

$3(1-\cos^2\theta) + 5\cos\theta = 1$

$3 - 3\cos^2\theta + 5\cos\theta = 1$

$0 = 3\cos^2\theta - 5\cos\theta - 2$

$-6 \;\; \times \; = -6$

$+ \; = -5$

$0 = 3\cos^2\theta - 6\cos\theta + 1\cos\theta - 2$

$0 = 3\cos\theta(\cos\theta - 2) + 1(\cos\theta - 2)$

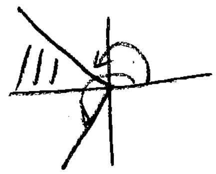
$0 = (\cos\theta - 2)(3\cos\theta + 1)$

$\cos\theta = 2$ $\quad$ $\cos\theta = -\frac{1}{3}$

No soln

$\cos\theta = -\frac{1}{3}$

ref



$\theta = \cos^{-1}\left(\frac{-1}{3}\right)$

$\theta_1 = 1.91$

$\text{ref}\angle = \pi - 1.91$

$\text{ref}\angle = 1.23$

$\theta_2 = \pi + \text{ref}\angle$

$\theta_2 = \pi + 1.23$

$\theta_2 = 4.37$

4. Find the intercepts, vertical asymptotes and points of discontinuity for the following functions:

a) $f(x) = \frac{x^2-x-6}{x^2-9}$

$f(x) = \frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x+3)}$

$f(x) = \frac{x+2}{x+3}$

vertical asymptote $x = -3$

point of discontinuity $x = 3$

$f(3) = \frac{3+2}{3+3} = \frac{5}{6}$ $\quad (3, 5/6)$

x-int

$0 = \frac{x+2}{x+3}$

$0 = x+2$

$-2 = x$ $\quad (-2, 0)$

y-int

$y = \frac{0+2}{0+3}$

$y = 2/3$

$(0, 2/3)$

b) $g(x) = \frac{x}{x^2-9}$

$g(x) = \frac{x}{(x-3)(x+3)}$

vertical asymptotes

$x = 3$ and $x = -3$

x-int

$0 = \frac{x}{x^2-9}$

$0 = x$

$(0,0)$

y-int

$y = \frac{0}{0-9}$

$y = \frac{0}{-9}$

$y = 0$

$(0,0)$

5. Prove the identity

$$\frac{tan2\theta(1 - tan\theta)cos^2\theta}{sin2\theta} = \frac{1}{1 + tan\theta}$$

$$\frac{\frac{2\tan\theta}{1-\tan^2\theta} \cdot \frac{(1-\tan\theta)}{1} \frac{\cos^2\theta}{1}}{2\sin\theta\cos\theta} = R.S.$$

$$\frac{\frac{2\tan\theta}{(1-\cancel{\tan\theta})(1+\tan\theta)} \cdot \frac{\cancel{(1-\tan\theta)}}{1} \cdot \frac{\cos^2\theta}{1}}{\frac{2\sin\theta\cos\theta}{1}} = R.S$$

$$\frac{\cancel{2}\tan\theta \cos^{\cancel{2}}\theta}{(1+\tan\theta)} \cdot \frac{1}{\cancel{2}\sin\theta\cancel{\cos\theta}} = R.S$$

$$\frac{\tan\theta\cos\theta}{(1+\tan\theta)\sin\theta} = R.S$$

$$\frac{\tan\theta}{1} \cdot \frac{\cos\theta}{(1+\tan\theta)\sin\theta} = R.S$$

$$\frac{\cancel{\sin\theta}}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{(1+\tan\theta)\cancel{\sin\theta}} = R.S$$

$$\frac{1}{1+\tan\theta} = \frac{1}{1+\tan\theta}.$$

Answers

Multiple Choice:

1.	B	11.	A	21.	A	31.	B	41.	A
2.	B	12.	C	22.	C	32.	D	42.	C
3.	D	13.	D	23.	C	33.	C		
4.	A	14.	A	24.	C	34.	D		
5.	A	15.	A	25.	D	35.	D		
6.	D	16.	D	26.	B	36.	B		
7.	D	17.	C	27.	D	37.	A		
8.	A	18.	B	28.	B	38.	D		
9.	B	19.	B	29.	B	39.	B		
10.	A	20.	C	30.	C	40.	B		

Written:

1. Some points on the new graph are (3,1) (0,-3) (-2,3) [redacted] (-5,3)

2. x = -2

3. $\theta = 1.91 \quad \theta = 4.37$

4.

	$f(x)$	$g(x)$
Vertical Asymptote	$x = -3$	$x = 3 \ x = -3$
Point of Discontinuity	$\left(3, \frac{5}{6}\right)$	None
x-intercept	$(-2,0)$	$(0,0)$
y-intercept	$\left(0, \frac{2}{3}\right)$	$(0,0)$