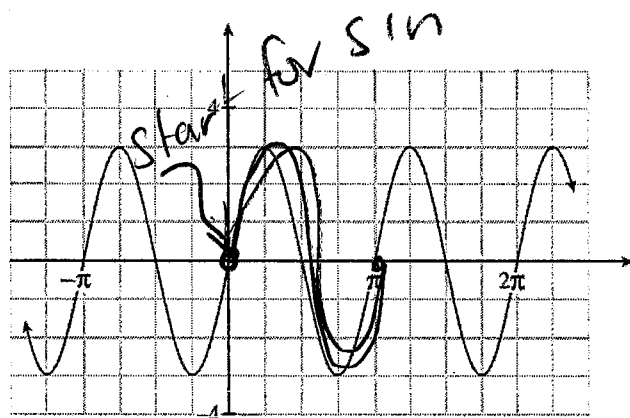


Pre-Calculus 12 Practice Exam B

Multiple-Choice: Part 1 Calculator Not Permitted

1. The graph of $y = a \sin bx$ is shown below. Determine the values of a and b .



$$\text{Amp} = \frac{|3 - (-3)|}{2} = 3$$

$$a = 3$$

$$\text{period} = \pi$$

$$b = \frac{2\pi}{\pi}$$

$$b = 2$$

A. $a = -3; b = 2$

B. $a = -3; b = \pi$

C. $a = 3; b = 2$

D. $a = 3; b = \pi$

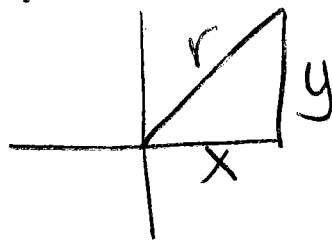
2. The terminal arm of angle θ in standard position intersects the unit circle at the point (x, y) . Which expression represents $\cot \theta$?

A. $\frac{y}{x}$

B. $\frac{x}{y}$

C. x

D. y



$$\tan \theta = \frac{y}{x}$$

$$\cot \theta = \frac{x}{y}$$

3. Which expression represents the measure of all angles in radians that are coterminal with angle θ ?

A. $2\pi + n\theta, n \in I$

B. $\theta + \frac{\pi}{2}n, n \in I$

C. $\theta + \pi n, n \in I$

D. $\theta + 2\pi n, n \in I$

Coterminal
add or subtract
multiples of
 2π or 360°

4. Determine the range of the function $y = -5\sin 2x - 3$.

A. $-8 \leq y \leq 2$

B. $-8 \leq y \leq -2$

C. $-5 \leq y \leq 5$

D. $-2 \leq y \leq 8$

Max = $1(5) - 3$

Max = 2

Min = $(-1)(5) - 3$

Min = -8

5. Determine the general solution for $\sin 4x = -1$.

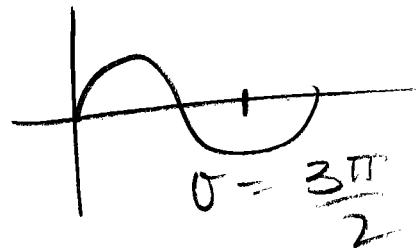
A. $x = \frac{\pi}{8} + \frac{\pi n}{2}$ where n is an integer

B. $x = \frac{3\pi}{8} + \frac{\pi n}{2}$ where n is an integer

C. $x = \frac{\pi}{8} + 2\pi n$ where n is an integer

D. $x = \frac{3\pi}{8} + 2\pi n$ where n is an integer

$4x = \theta$
 $\sin \theta = -1$



$\frac{3\pi}{2} = 4x$

$\frac{3\pi}{8} = x$

period = $\frac{2\pi}{4} = \frac{\pi}{2}$

So A or B

6. Chantal simplified the expression $\frac{\csc \theta + \sec \theta}{\sin \theta + \cos \theta}$ as shown below. In which step is Chantal's first error?

Steps	
1.	$\frac{1\sqrt{\quad} + 1\sqrt{\quad}}{\sin \theta + \cos \theta}$
2.	$\frac{\cos \theta + \sin \theta}{\sin \theta + \cos \theta}$
3.	$\left(\frac{\cos \theta + \sin \theta}{\sin \theta}\right)\left(\frac{1}{\sin \theta + \cos \theta}\right)$
4.	$\frac{1}{\sin \theta}$

~~$\frac{\cos \theta + \sin \theta}{\sin \theta \cos \theta} + \frac{\sin \theta}{\sin \theta \cos \theta}$~~
 ~~$\frac{\sin \theta + \cos \theta}{\sin \theta + \cos \theta}$~~

- A. 1
 B. 2
 C. 3
 D. 4

7. Determine all non-permissible values for the expression $\frac{\sec x}{2 \sin x + 1}$, in the interval $0 \leq x < 2\pi$

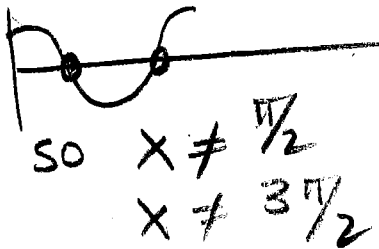
A. $x = \frac{\pi}{2}, \frac{3\pi}{2}$

B. $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

C. $x = 0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$

D. $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

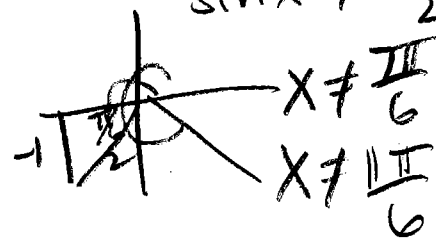
sec x has asymptotes when $\cos x = 0$



$2 \sin x + 1 \neq 0$

$2 \sin x \neq -1$

$\sin x \neq -\frac{1}{2}$



8. The function $h = -5 \cos \frac{\pi}{60} t + 6$ gives Cassandra's height, h metres, above the ground when she is riding a Ferris wheel after t seconds. Determine the length of time for one rotation and the lowest point on the Ferris wheel.

- A. 60 sec, 6 m
 B. 60 sec, 1 m
 C. 120 sec, 6 m
 D. 120 sec, 1 m

1 rotation = 1 period

$B = \frac{\pi}{60}$

period = $\frac{2\pi}{B}$

$= \frac{2\pi}{\frac{\pi}{60}} = 2\pi \cdot \frac{60}{\pi} = 120$

Min = $-1(5) + 6$
 Min = 1

9. Determine the number of solutions for $\sin^2 x (\csc x + 1) = 0$ in the interval $0 \leq x < 2\pi$ with the correct reasoning.

$\sin x = 0$  $\csc x = -1$
 $\sin x = -1$

A. There are three solutions because $\sin^2 x = 0$ has two solutions and $\csc x + 1 = 0$ has one solution in the interval $0 \leq x < 2\pi$.

B. There are three solutions because $\sin^2 x = 0$ has one solution and $\csc x + 1 = 0$ has two solutions in the interval $0 \leq x < 2\pi$.

C. There is one solution because $\sin^2 x = 0$ has to be rejected and $\csc x + 1 = 0$ has one solution in the interval $0 \leq x < 2\pi$.

D. There is one solution because $\sin^2 x = 0$ has one solution and $\csc x + 1 = 0$ has to be rejected in the interval $0 \leq x < 2\pi$.

1
 However $\csc x$ has asymptotes when $\sin x = 0$ so can't have the soln from \sin

10. Two students, Yuri and Rubin, solved the exponential equation $2^{x+1} = 3$ as shown below.

Yuri's Solution	Rubin's Solution
$2^{x+1} = 3$	$2^{x+1} = 3$
$\log 2^{x+1} = \log 3$	$x+1 = \log_2 3$
$(x+1)\log 2 = \log 3$	$x = \log_2 3 - 1$
$x \log 2 + \log 2 = \log 3$	
$x \log 2 = \log 3 - \log 2$	
$x = \frac{\log 3 - \log 2}{\log 2}$	

$c^m = b$
 $\log_c b = m$

Which statement is true?

A. Yuri is incorrect, Rubin is incorrect.

B. Yuri is incorrect, Rubin is correct.

C. Yuri is correct, Rubin is incorrect.

D. Yuri is correct, Rubin is correct.

11. What is the best estimation of $\log_3 30$?

- A. 3.1
- B. 3.4
- C. 3.6
- D. 3.9

$$\log_3 27$$

$$\log_3 3^3$$

$$3$$

$$\log_3 81$$

$$\log_3 3^4$$

$$4$$

12. Solve for x : $\log_2 3 = 2 \log_8 x$

- A. $3^{\frac{2}{3}}$
- B. $3^{\frac{3}{2}}$
- C. $2^{\frac{2}{3}}$
- D. $2^{\frac{3}{2}}$

$$\log_2 3 = 2 \frac{\log_2 x}{\log_2 8}$$

$$\log_2 3 = \frac{2 \log_2 x}{3}$$

$$\log_2 3 = \frac{2}{3} \log_2 x$$

$$\log_2 3 = \log_2 x^{2/3}$$

$$3 = x^{2/3}$$

$$3^{3/2} = (x^{2/3})^{3/2}$$

$$3^{3/2} = x$$

13. Determine the Richter scale reading for an earthquake that is 5 times more intense than another earthquake that measures 4.0 on the Richter scale.

- A. 9
- B. 20
- C. $4 + \log 5$
- D. $5 + \log 4$

Strong x

$$x = \log \frac{A}{A_0}$$

$$10^x = \frac{A}{A_0}$$

Weak 4

$$4 = \log \frac{A}{A_0}$$

$$10^4 = \frac{A}{A_0}$$

$$\frac{10^x}{10^4} = 5$$

$$10^{x-4} = 5$$

$$\log 5 = x - 4$$

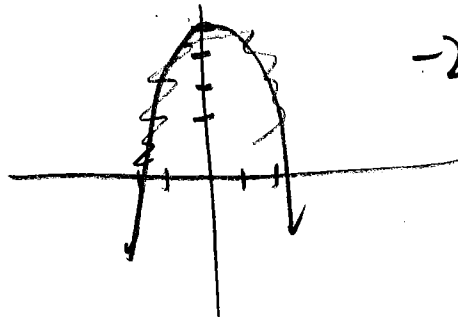
$$\log 5 + 4 = x$$

14. Determine the domain of the function $y = \log(4 - x^2)$.

- A. $-2 < x < 2$
- B. $-2 \leq x \leq 2$
- C. $x < -2, x > 2$
- D. $x \leq -2, x \geq 2$

$$4 - x^2 > 0$$

$$-2 < x < 2$$

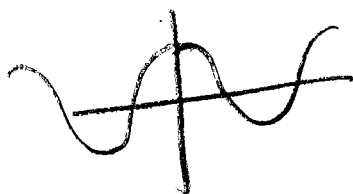


15. Explain how the graph of $y - 5 = f(x)$ is related to the graph of $y = f(x)$.

- A. It is the graph of $y = f(x)$ translated 5 units up.
- B. It is the graph of $y = f(x)$ translated 5 units down.
- C. It is the graph of $y = f(x)$ translated 5 units to the left.
- D. It is the graph of $y = f(x)$ translated 5 units to the right.

$$y = f(x) + 5$$

16. For which of the following functions is $f(x) = f(-x)$?



I.	$y = \sin x$
II.	$y = \cos x$
III.	$y = (x-3)^2$
IV.	$y = x^2 + 3$

reflection over
y-axis
give no change
to graph



- A. I, III only
- B. I, IV only
- C. II, III only
- D. II, IV only

17. The point $P(-3, -8)$ is on the graph of $y = f(x)$. Which point must be on the graph of $y = -f(x-5)$?

- A. $(-8, -8)$
- B. $(-8, 8)$
- C. $(2, 8)$
- D. $(8, -8)$

$a = -1$
mult y's by (-1)
translate right 5
add 5 to x's

$$(-3, -8) \rightarrow (2, 8)$$

18. The graph of $y = f(x)$ is shown below. Which graph represents $y = f(2x - 4)$.

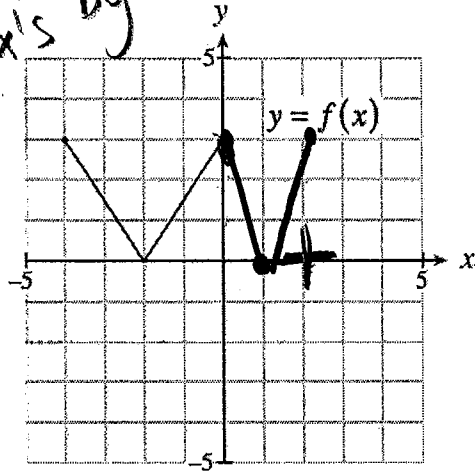
old

$$\begin{array}{r|l} -4 & 3 \\ -2 & 0 \\ 0 & 3 \end{array}$$

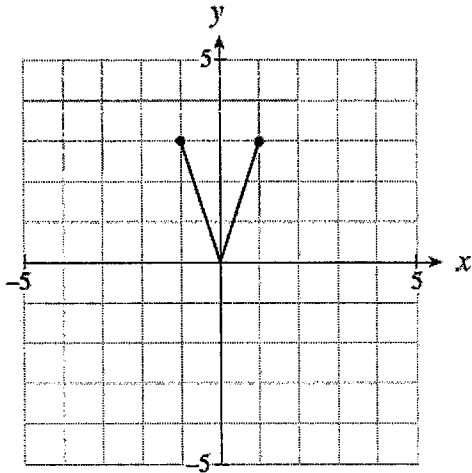
$b = 2$
divide x's by 2

$$\begin{array}{r|l} -2 & 3 \\ -1 & 0 \\ 0 & 3 \end{array}$$

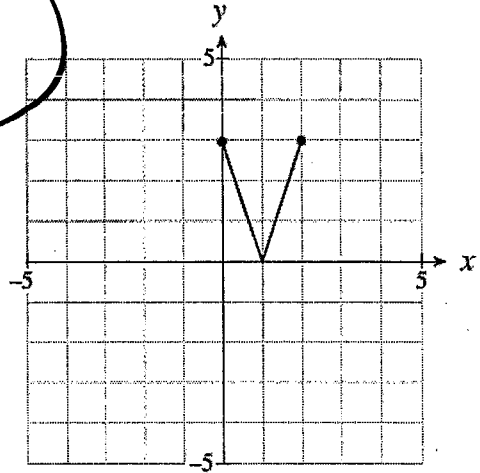
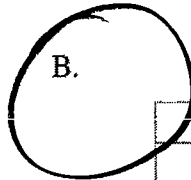
$$y = f(2(x - 2))$$



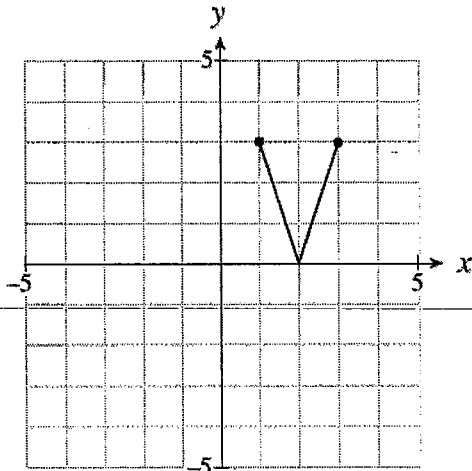
A.



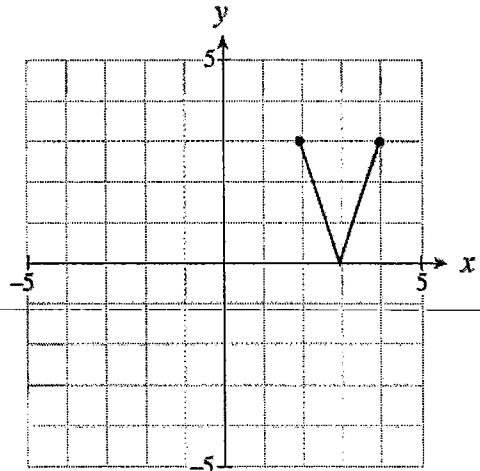
B.



C.

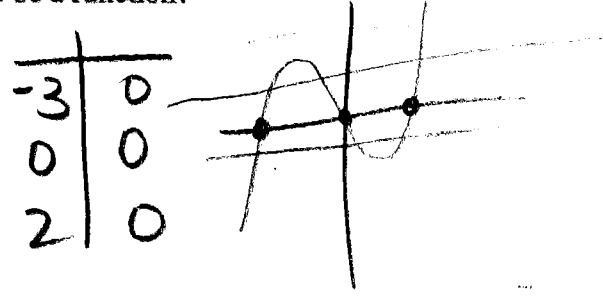


D.



19. A cubic polynomial function f has zeros $\{-3, 0, 2\}$. Which restriction on the domain of f will allow its inverse to be a function?

- A. $x > -3$
- B. $x > 0$
- C. $x < 0$
- D. $x > 2$



20. Raj used synthetic division to divide a polynomial $f(x)$ by $x-2$ as shown below.

$$\begin{array}{r|rrrr} 2 & 1 & -3 & k & -5 \\ & & 2 & -4 & 2k-10 \\ \hline & 1 & -1 & k-4 & 2k-15 \end{array}$$

$f(x) = 1x^3 - 3x^2 + kx - 5$
 $f(2) = -1$

Determine the value of k that will give a remainder of -1 as shown in the table.

- A. 1
- B. 4
- C. 5
- D. 6

$$\begin{aligned} 2^3 - 3(2)^2 + k(2) - 5 &= -1 \\ 8 - 12 + 2k - 5 &= -1 \\ 2k - 9 &= -1 \\ 2k &= 8 \\ k &= 4 \end{aligned}$$

21. Compare the graphs of the two functions at $x = 2$.

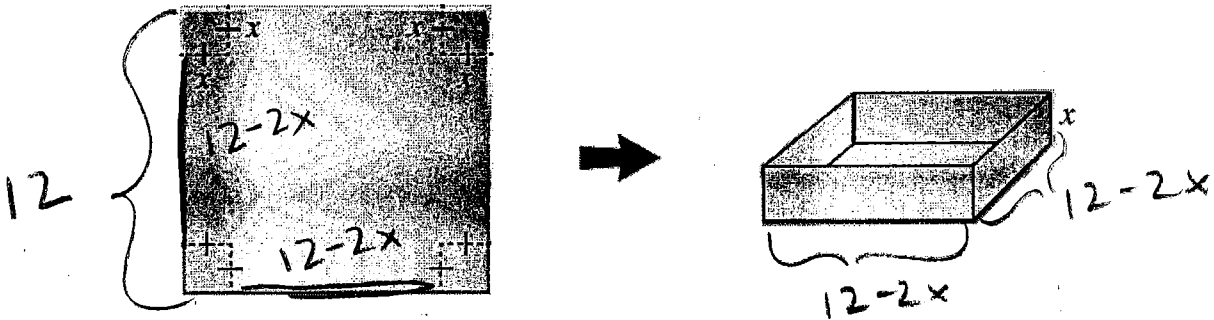
$$f(x) = x(x-2)^3(x+2) \text{ and } g(x) = x(x-2)^2(x+2)$$

- A. The graph of $f(x)$ crosses the x -axis at $x = 2$ and the graph of $g(x)$ just touches the x -axis at $x = 2$ but does not cross it.
- B. The graph of $f(x)$ just touches the x -axis at $x = 2$ but does not cross it and the graph of $g(x)$ crosses the x -axis at $x = 2$.
- C. The graph of $f(x)$ crosses the x -axis at $x = 2$ and the graph of $g(x)$ crosses the x -axis at $x = 2$.
- D. The graph of $f(x)$ just touches the x -axis at $x = 2$ but does not cross it and the graph of $g(x)$ just touches the x -axis at $x = 2$ but does not cross it.

$f(x)$
 $x = 0$
 $x = 2$ mult 3 goes through x -axis
 $x = -2$

$g(x)$
 $x = 0$
 $x = 2$ mult 2 touches x -axis
 $x = -2$

36. A sheet of metal $12\text{ cm} \times 12\text{ cm}$ will be used to make an open-top box by removing a square of length x in each corner and turning up the sides as shown in the diagram.



What is the volume of the box as a function of x ?

- A. $V = x^3$
 B. $V = x(12-x)^2$
 C. $V = 144 - 4x^2$
 D. $V = x(12-2x)^2$

37. Determine all solutions for the equation $\sqrt{x+4} = 3x$.

- A. $-0.61, 0.72$
 B. -0.61
 C. 0.72
 D. 1.33

$$\begin{aligned} x+4 &= 9x^2 \\ (\sqrt{x+4})^2 &= (3x)^2 \\ x+4 &= 9x^2 \\ 0 &= 9x^2 - x - 4 \end{aligned}$$

$$\begin{aligned} x &= \frac{-1 \pm \sqrt{(-1)^2 - 4(9)(-4)}}{2(9)} \\ x &= \frac{-1 \pm \sqrt{1 + 144}}{18} \\ x &= \frac{-1 \pm \sqrt{145}}{18} \end{aligned}$$

38. Which equation represents the graph of $f(x)$ after it has been horizontally stretched by a factor of $\frac{1}{2}$?

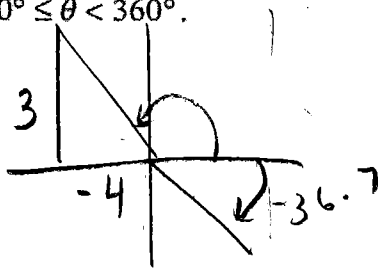
- A. $y = f(2x)$
 B. $y = 2f(x)$
 C. $y = f\left(\frac{1}{2}x\right)$
 D. $y = \frac{1}{2}f(x)$

Horizontal stretch
 factor of $\frac{1}{2}$
 $b = 2$

~~$x = -0.61$~~
 $x = 0.72$
 does not work in the check.

32. Determine the measure of the standard position angle θ if the point $P(-4, 3)$ is on the terminal arm of angle θ , where $0^\circ \leq \theta < 360^\circ$.

- A. 37°
 B. 53°
 C. 127°
 D. 143°



$$\tan \theta = \frac{3}{-4}$$

$$\theta = \tan^{-1}\left(\frac{-3}{4}\right)$$

$$\theta = -37^\circ$$

$$\text{ref} = 37 \quad \theta = 180 - 37$$

33. Express as a single logarithm: $\log a - \log b - 3 \log c$

A. $\log \frac{a}{bc^3}$

B. $\log \frac{a}{b^3c^3}$

C. $\log \frac{ac^3}{b}$

D. $\log \frac{ac^3}{b^3}$

$$\log a - \log b - \log c^3$$

$$\log \frac{a}{bc^3}$$

34. Determine the x -intercept of the function $y = 5^x - 3$.

- A. -2
 B. 0.008
 C. 0.6
 D. 0.68

$$y = 0$$

$$0 = 5^x - 3$$

$$3 = 5^x$$

$$\log 3 = \log 5^x$$

$$\log 3 = x \log 5$$

$$\frac{\log 3}{\log 5} = x \quad x = .68$$

35. An investment earns 2.25% per annum compounded daily. How many years would be required for an investment to triple in value? Assume all years have 365 days.

- A. 4.88
 B. 5.41
 C. 48.83
 D. 49.37

$$P = 1$$

$$A = 3$$

$$r = .0225$$

$$n = 365$$

$$t = ?$$

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$3 = 1 \left(1 + \frac{.0225}{365}\right)^{365t}$$

$$3 = 1.000061644^{365t}$$

$$\log 3 = \log (1.000061644)^{365t}$$

$$\log 3 = 365t \log (1.000061644)$$

$$\frac{\log 3}{365 \log 1.000061644} = t$$

29. For the function $f(x) = \frac{x^2 - 4}{x^2 - 2x}$, which of the following statements explain the behaviour of the graph of f for the values of a variable near a non-permissible value?

- A. When x is close to 2 on either side, f is close to 2.
When x is just to the right of 0, f is a large positive value.
When x is just to the left of 0, f is a large negative value.
- B. When x is close to 2 on either side, f is close to 4.
When x is just to the right of 0, f is a large positive value.
When x is just to the left of 0, f is a large negative value.
- C. When x is close to 2 on either side, f is close to 2.
When x is just to the right of 0, f is a large negative value.
When x is just to the left of 0, f is a large positive value.
- D. When x is close to 2 on either side, f is close to 4.
When x is just to the right of 0, f is a large negative value.
When x is just to the left of 0, f is a large positive value.

$$f(x) = \frac{(x-2)(x+2)}{x(x-2)} = \frac{x+2}{x}$$

$x=2$ point of discontinuity

$$f(2) = \frac{2+2}{2} = \frac{4}{2} = 2$$

Hole (2, 2)

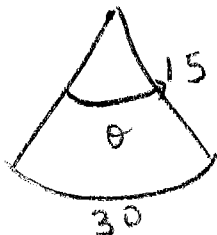
$$f(x) = \frac{x+2}{x} = \frac{x}{x} + \frac{2}{x} = 1 + \frac{2}{x}$$



Multiple-Choice: Part 2 Calculator Permitted

30. A pendulum 15cm long swings through an arc of length 30cm. Through what angles does the pendulum swing? (Answers accurate to the nearest degree.)

- A. 2°
B. 60°
C. 75°
D. 115°



$$\frac{\theta}{360^\circ} = \frac{30}{2\pi(15)}$$

$$\theta = \frac{30(360)}{2\pi(15)}$$

31. Determine the equation of a circle with centre (0, 0) passing through the point P(-2, 5).

- A. $x^2 + y^2 = 3$
B. $x^2 + y^2 = 9$
C. $x^2 + y^2 = 21$
D. $x^2 + y^2 = 29$

$$x^2 + y^2 = r^2$$

$$(-2)^2 + (5)^2 = r^2$$

$$4 + 25 = r^2$$

$$r^2 = 29$$

27. Determine the equations of all asymptotes for the graph of $y + 2 = \frac{1}{x-1}$.

A. $x = -1, y = -2$

B. $x = -1; y = 2$

C. $x = 1, y = -2$

D. $x = 1, y = 2$

$$y = \frac{1}{x-1} - 2$$

$x = 1$
vertical asymptote

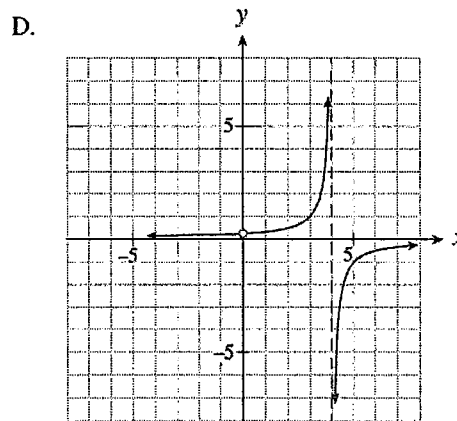
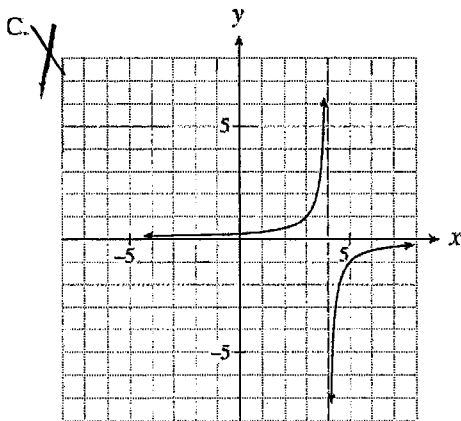
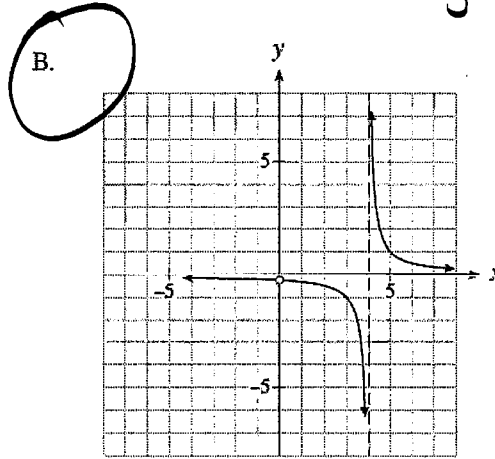
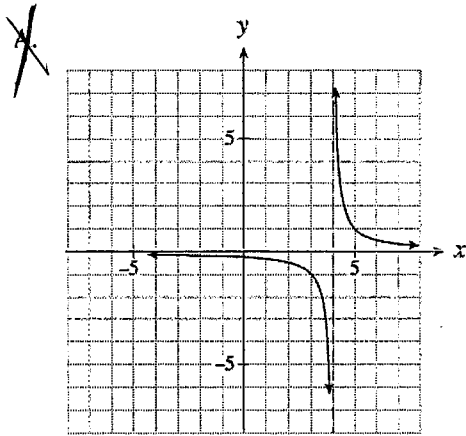
$y = -2$
horizontal asymptote

28. Which of the following best represents the graph of the rational function $y = \frac{x}{x^2 - 4x}$?

$$y = \frac{x}{x(x-4)} = \frac{1}{x-4}$$

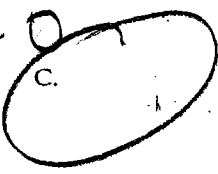
point of discontinuity at $x = 0$

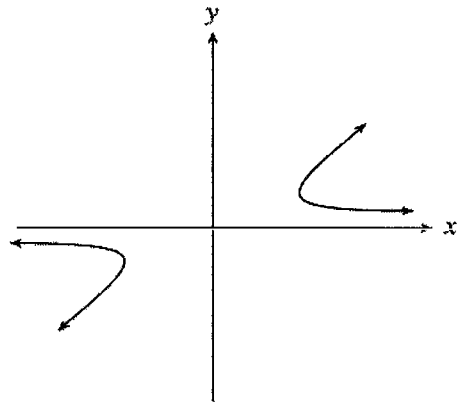
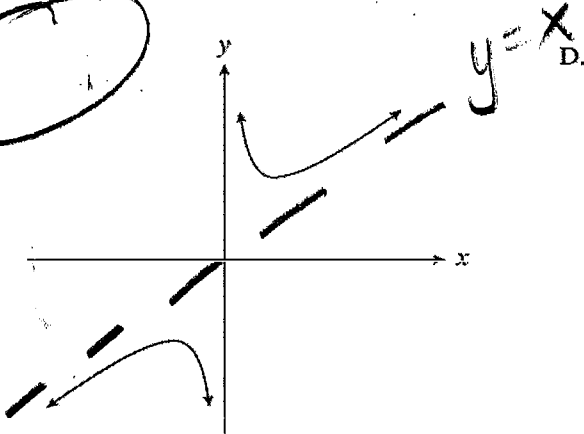
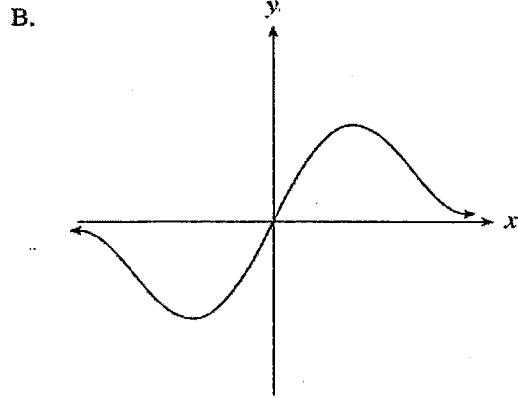
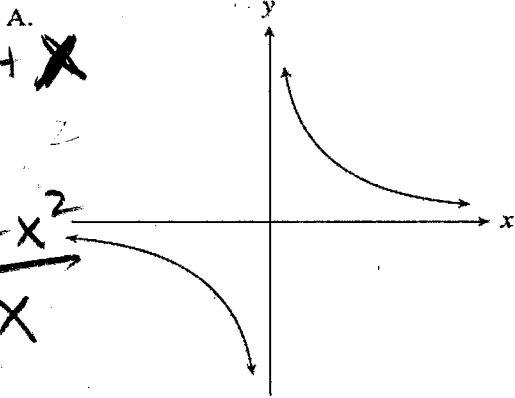
vertical asymptote at $x = 4$



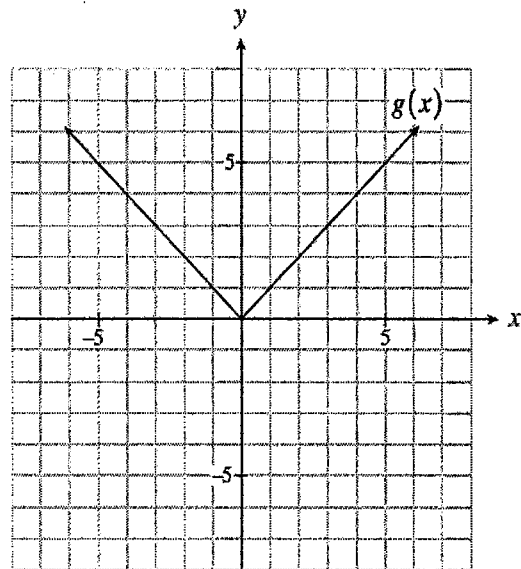
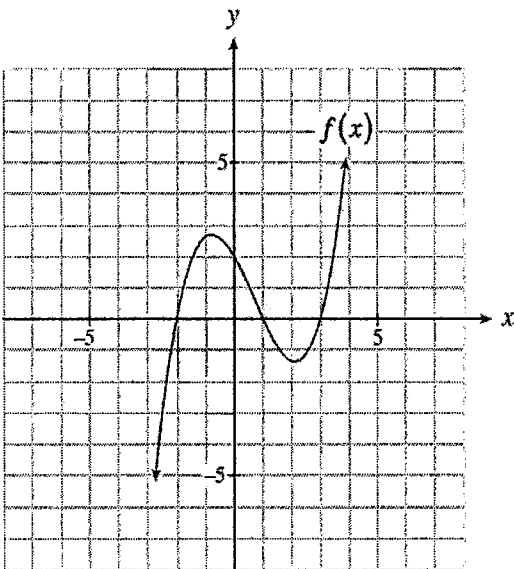
25. Given $f(x) = x$ and $g(x) = \frac{1}{x}$, which graph best represents $y = f(x) + g(x)$?

$y = \frac{1}{x} + x$
 or
 $y = \frac{1+x^2}{x}$

$x \neq 0$




26. The graphs of $f(x)$ and $g(x)$ are given below. Determine $f(g(-3))$.



- A. -6
- B. -2
- C. 0**
- D. 3

$g(-3) = 3$
 $f(g(-3)) = f(3) = 0$

22. Given the functions $f(x) = x + 3$ and $g(x) = x^2 - 4$, determine the value of $(f + g)(-2)$.

- A. 0
 B. 1
 C. 3
 D. 5

$$f(2) = -2 + 3$$

$$f(-2) = 1$$

$$g(-2) = (-2)^2 - 4$$

$$g(-2) = 4 - 4 = 0$$

23. For which of the following functions is $f(f(x)) = x$, for all values of x in the domain of f ?

I.	$f(x) = x$
II.	$f(x) = -x$
III.	$f(x) = \frac{1}{x}$

$$f(f(x)) = x$$

$$f(-x) = -(-x) = x$$

$$f\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = 1 \cdot \frac{x}{1} = x$$

- A. I and II only
 B. I and III only
 C. II and III only
 D. I, II and III

24. A polynomial function f has zeros at 1, -1, and 2. Given the function $g(x) = \frac{x+1}{x-2}$,

determine the domain of the function $h(x) = \frac{f(x)}{g(x)}$.

- A. all real numbers
 B. all real numbers, $x \neq 2$
 C. all real numbers, $x \neq -1$

- D. all real numbers $x \neq -1$ and $x \neq 2$

$$f(x) = (x-1)(x+1)(x-2)$$

$$h(x) = \frac{(x-1)(x+1)(x-2)}{\frac{x+1}{x-2}}$$

$$x \neq -1$$

$$x \neq 2$$

39. Evaluate: $\sum_{k=1}^4 \cos \frac{k\pi}{4}$

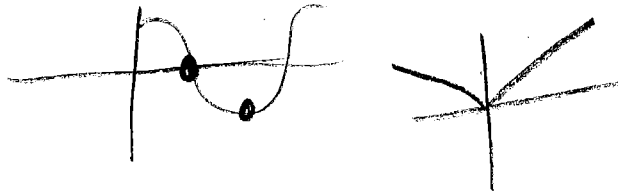
A. -1

B. 0

C. 1

D. $\frac{4}{\sqrt{2}}$

$$\begin{aligned} & \cos \frac{\pi}{4} + \cos \frac{2\pi}{4} + \cos \frac{3\pi}{4} + \cos \frac{4\pi}{4} \\ & \cos \frac{\pi}{4} + \cos \frac{\pi}{2} + \cos \frac{3\pi}{4} + \cos \pi \\ & \frac{1}{\sqrt{2}} + 0 + \left(-\frac{1}{\sqrt{2}}\right) + (-1) \\ & = -1 \end{aligned}$$



40. Determine an expression for the number of terms in the series:

$$\sum_{k=a}^b 5(3)^{k-1}$$

$$n = b - a + 1$$

A. b

B. $b - a$

C. $b - a + 1$

D. $b - a - 1$

41. Determine the sum of the first 12 terms of the geometric series: $-18 + 12 - 8 \dots$

A. -941.37

B. -10.88

C. -10.72

D. 926.97

$$b_1 = -18$$

$$r = \frac{12}{-18}$$

$$r = -\frac{2}{3}$$

$$n = 12$$

$$S_{12} = \frac{-18 \left(\left(-\frac{2}{3} \right)^{12} - 1 \right)}{-\frac{2}{3} + 1}$$

$$S_{12} = -18 \left(\dots \right)$$

42. In a geometric sequence, $t_4 = -192$ and $t_7 = 12288$. Determine the first term.

A. -4

B. -3

C. 3

D. 4

$$\begin{aligned} -192 &= t_1(r)^{4-1} & 12288 &= t_1(r)^{7-1} \\ -192 &= t_1 r^3 & 12288 &= t_1 r^6 \end{aligned}$$

$$\begin{aligned} \frac{12288}{-192} &= \frac{t_1 r^6}{t_1 r^3} \\ -64 &= r^3 \\ r &= -4 \end{aligned}$$

$$\begin{aligned} -192 &= t_1(-4)^3 \\ -192 &= t_1(-64) \\ \frac{-192}{-64} &= t_1 \\ t_1 &= 3 \end{aligned}$$

43. A shoe store is closing and wants to sell all of its shoes. At the start of each week, the price of all shoes is reduced by 10% of the current price. If a pair of shoes costs \$100 during the first week of the sale, determine the price of these shoes during the 6th week of the sale.

A. \$50.00

B. \$53.14

C. \$59.05

D. \$65.61

$$100, 100(.9), 100(.9)^2, \dots$$

$$\begin{aligned} t_1 &= 100 \\ r &= 100\% - 10\% \\ r &= 90\% = 0.9 \\ n &= 6 \end{aligned}$$

$$\begin{aligned} t_6 &= 100(.9)^6 \\ t_6 &= 100(.9)^5 \\ t_6 &= 59.05 \end{aligned}$$

44. Determine the common ratio of the infinite geometric series:

$$\log_3 a + \log_9 a + \log_{81} a + \dots \text{ where } a > 0$$

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. 2

D. 3

$$r = \frac{\log_9 a}{\log_3 a}$$

$$r = \frac{\log_3 a}{\log_3 a} = 1$$

change of base

$$r = \frac{\log_3 a}{2 \log_3 a} = \frac{1}{2}$$

45. Solve: $2\ln(x+3) = 12$

A. 5.158

B. 20.086

C. 400.429

D. 162 751.791

$$\ln(x+3) = \frac{12}{2}$$

$$\ln(x+3) = 6$$

$$e^6 = x+3$$

$$e^6 - 3 = x$$

$$x = 400.429$$

46. Solve: $e^{3x+2} = 18$

A. -0.248

B. 0.297

C. 2.224

D. 10.671

$$\ln e^{3x+2} = \ln 18$$

$$(3x+2) \ln e = \ln 18$$

$$3x+2 = \ln 18$$

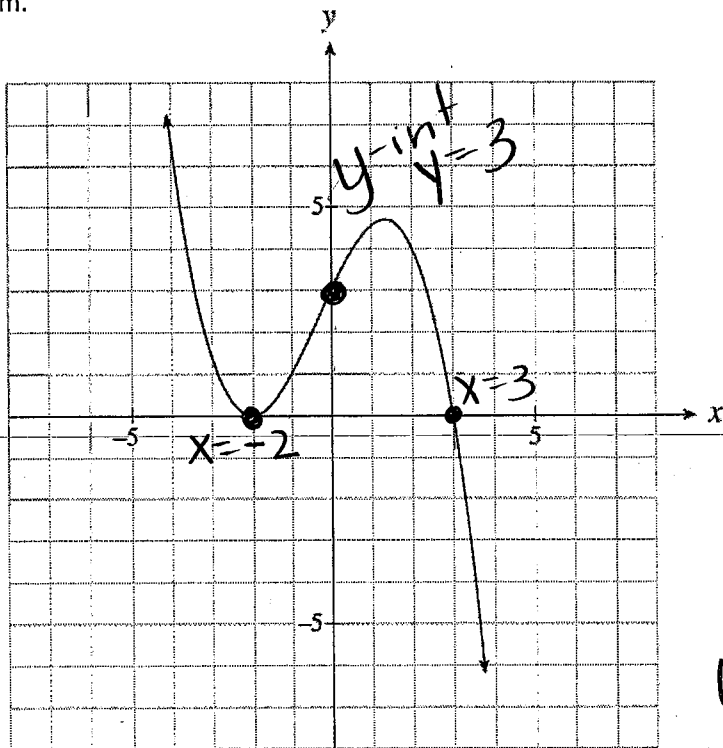
$$3x = \ln 18 - 2$$

$$x = \frac{\ln 18 - 2}{3}$$

$$x =$$

Written Response:

1. Determine an equation for the cubic polynomial function graphed below. Leave answer in factored form.



$$x = -2 \text{ Mult } 2$$

$$x = 3$$

$$y = a(x+2)^2(x-3)$$

$$y\text{-int } (0, 3)$$

$$3 = a(0+2)^2(0-3)$$

$$3 = a(4)(-3)$$

$$3 = -12a$$

$$-\frac{3}{12} = a$$

$$-\frac{1}{4} = a$$

$$y = -\frac{1}{4}(x+2)^2(x-3)$$

2. Find the inverse function for $f(x) = 2^x + 1$. Graph $f(x)$ and $f^{-1}(x)$ on the same grid.

$y = 2^x + 1$

Inverse

$$x = 2^y + 1$$

$$x - 1 = 2^y$$

$y = 2^x$ and up 1

-1	1/2
0	1
1	2
2	4

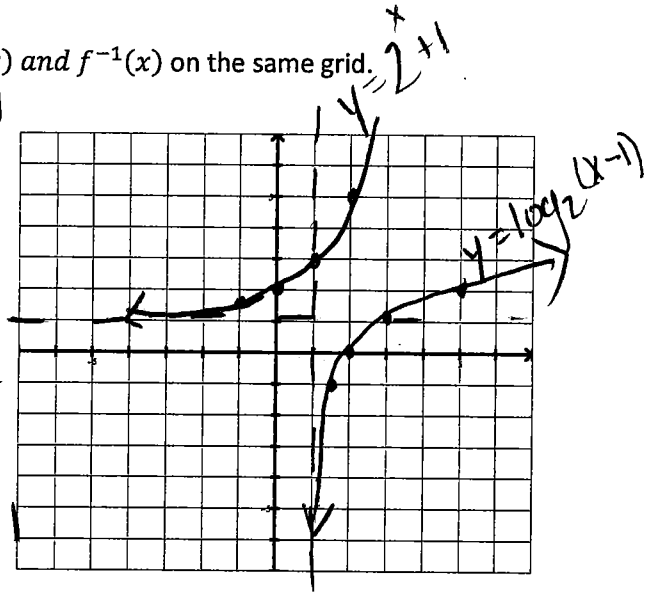
$\log_2(x-1) = y$

Vertical asymptote $x=1$

$y = \log_2 x$

$2^x = y$ and right 1

1/2	-1
1	0
2	1
4	2



3. A food sample contains 300 bacteria. The doubling time for bacteria left at room temperature is 20 minutes. How many minutes will it take to reach an unsafe level of 100 000 bacteria?

$A_0 = 300$

$C = 2$

$T = 20$

$A = 100\ 000$

$t = ?$

$A = A_0(C)^{t/T}$

$100\ 000 = 300(2)^{t/20}$

$\frac{1000}{3} = 2^{t/20}$

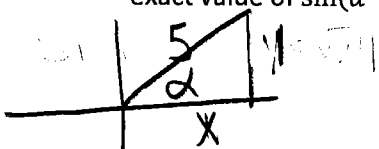
$\log \frac{1000}{3} = \log 2^{t/20}$

$\log \frac{1000}{3} = \frac{t}{20} \log 2$

$20 \log \frac{1000}{3} = t$

$t = 167.6 \text{ min}$

4. Given $\sin \alpha = \frac{1}{5}$, where α is in quadrant I and $\cos \beta = \frac{2}{3}$ where β is in quadrant IV, determine the exact value of $\sin(\alpha + \beta)$

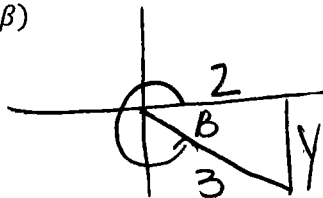


$x^2 + 1^2 = 5^2$

$x^2 + 1 = 25$

$x^2 = 24$

$x = \sqrt{24}$



$x^2 + 2^2 = 3^2$

$x^2 + 4 = 9$

$x^2 = 5$

$x = -\sqrt{5}$

$\sin(\alpha + \beta)$

$= \sin \alpha \cos \beta + \cos \alpha \sin \beta$

$= \left(\frac{1}{5}\right)\left(\frac{2}{3}\right) + \left(\frac{\sqrt{24}}{5}\right)\left(\frac{-\sqrt{5}}{3}\right)$

$= \frac{2}{15} - \frac{(2\sqrt{6})(\sqrt{5})}{15}$

$= \frac{2 - 2\sqrt{30}}{15}$

5. Prove algebraically:

$$\frac{\cos \theta}{1 - \sin \theta} = \sec \theta + \sec \theta \csc \theta - \cot \theta$$

$$L.S = \frac{1}{\cos \theta} + \frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta}$$

$$L.S = \frac{\sin \theta}{\sin \theta \cos \theta} + \frac{1}{\sin \theta \cos \theta} - \frac{\cos^2 \theta}{\sin \theta \cos \theta}$$

$$L.S = \frac{\sin \theta + 1 - \cos^2 \theta}{\sin \theta \cos \theta}$$

$$L.S = \frac{\sin \theta + \sin^2 \theta}{\sin \theta \cos \theta}$$

$$L.S = \frac{\sin \theta (1 + \sin \theta)}{\sin \theta \cos \theta}$$

$$L.S = \frac{1 + \sin \theta}{\cos \theta} \cdot \frac{(1 - \sin \theta)}{(1 - \sin \theta)}$$

$$L.S = \frac{1 - \sin^2 \theta}{\cos \theta (1 - \sin \theta)}$$

conjugate

Answers

Multiple Choice:

1. C	11. A	21. A	31. D	41. C
2. B	12. B	22. B	32. D	42. C
3. D	13. C	23. D	33. A	43. B
4. A	14. A	24. D	34. D	44. B
5. B	15. A	25. C	35. C	45. C
6. B	16. D	26. C	36. D	46. B
7. D	17. C	27. C	37. C	
8. D	18. B	28. B	38. A	
9. C	19. D	29. A	39. A	
10. D	20. B	30. D	40. C	

Written:

1. $y = \frac{-1}{4}(x+2)^2(x-3)$

2. $f^{-1}(x) = \log_2(x-1)$

3. 167.6 mins

4. $\frac{2-2\sqrt{30}}{15}$

$$L.S = \frac{1 - \sin^2 \theta}{\cos \theta (1 - \sin \theta)}$$

$$L.S = \frac{\cos^2 \theta}{\cos \theta (1 - \sin \theta)}$$

$$\frac{\cos \theta}{1 - \sin \theta} = \frac{\cos \theta}{1 - \sin \theta}$$