Related Rates Review

Thursday, November 2, 2017 2:03 PM

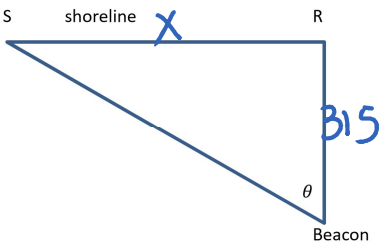
Related Rates Review

1. The radius of a spherical balloon is expanding at a rate of 14 in/min. At what rate is the volume changing when the radius is 8 inches.

$11259.5 \text{ in}^3/\text{min}$

$3584\pi \text{ in}^3/\text{min}$

2. A beacon, located on a perpendicular distance of 315m from point R on a straight shoreline, revolves at 1 rev/min. How fast does its beam sweep along the shoreline at point S on the shoreline 425m from R?



$\frac{1 \text{ rev}}{1 \text{ min}} = \frac{2\pi \text{ rads}}{1 \text{ min}}$

$\frac{d\theta}{dt} = 2\pi \text{ rad/min}$

$\tan\theta = \frac{x}{315}$

$315 \tan\theta = x$

$315 \sec^2\theta \frac{d\theta}{dt} = \frac{dx}{dt}$

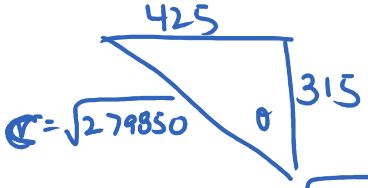
$315\left[\frac{\sqrt{279850}}{315}\right]^2 (2\pi) = \frac{dx}{dt}$

$\frac{279850}{315}(2\pi) = \frac{dx}{dt}$

$\frac{dx}{dt} = 5582 \text{ m/min}$

5582 m/min

when $x = 425$



$425^2 + 315^2 = c^2$

$c = \sqrt{279850}$

$\sec\theta = \frac{\sqrt{279850}}{315}$

3. Sand is being dumped from a conveyor belt at a rate of 1.2 m^3/min and forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile growing when the pile is 3m high?

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$\frac{dV}{dt} = 1.2$

$d = h \qquad r = \frac{h}{2}$

$2r = h$

$h = 3$

.17 m/min

$\frac{8}{15\pi}$ m/min

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$V = \frac{1}{3}\pi\left(\frac{h^2}{4}\right)h$

$V = \frac{\pi}{12}h^3$

$\frac{dV}{dt} = \frac{\pi}{12}(3h^2)\frac{dh}{dt}$

$d = h \qquad r = \frac{h}{2}$

$2r = h$

$\frac{dh}{dt} = ?$ when $h = 3$

$1.2 = \frac{\pi}{4}(3)^2\frac{dh}{dt}$

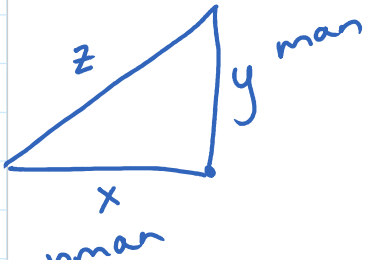
$\frac{1.2(4)}{9\pi} = dh/dt$

$\frac{8}{15\pi}$ m/min

$\frac{8}{15\pi}$ ✓

.17

4. A man is walking north at a speed of 1.5 m/s and a woman starts at the same point P at the same time walking west at a speed of 2 m/s. At what rate is the distance between the man and the woman increasing one minute later?



$\frac{dy}{dt} = 1.5$ m/s $\qquad \frac{dx}{dt} = 2$ m/s

$y = 1.5(60) \qquad x = 2(60)$

$y = 90 \qquad x = 120$

Ans: 2.5 m/s

$\frac{dz}{dt} = ?$

$t = 1$ min

$t = 60$ sec

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$\frac{dz}{dt} = 2.5$ m/s