

Substitution Review

Wednesday, January 18, 2017 9:04 AM

AP Calculus Substitution Review

1. $\int x(x^2 - 3)^6 dx$

$$u = x^2 - 3$$

$$\frac{du}{dx} = 2x$$

$$\frac{du}{2} = x dx$$

$$\begin{aligned} & \int u^6 \cdot \frac{du}{2} \\ &= \frac{1}{2} \int u^6 du \\ &= \frac{1}{2} \cdot \frac{1}{7} u^7 + C \\ &= \frac{1}{14} (x^2 - 3)^7 + C \end{aligned}$$

2. $\int 2 \cos 5x dx$

$$u = 5x$$

$$\frac{du}{dx} = 5$$

$$\frac{du}{5} = dx$$

$$\begin{aligned} &= 2 \int \cos u \cdot \frac{du}{5} \\ &= \frac{2}{5} \sin u + C \\ &= \frac{2}{5} \sin 5x + C \end{aligned}$$

3. $\int x(x+5)^3 dx$

$$u = x + 5$$

$$\frac{du}{dx} = 1$$

$$du = dx$$

$$u = x + 5$$

$$u - 5 = x$$

$$\begin{aligned} & \int (u-5) u^3 du \\ &= \int u^4 - 5u^3 du \\ &= \frac{1}{5} u^5 - \frac{5}{4} u^4 + C \\ &= \frac{4u^5}{20} - \frac{25u^4}{20} + C \\ &= \frac{u^4}{20} [4u - 25] + C \\ &= \frac{(x+5)^4}{20} [4(x+5) - 25] + C \end{aligned}$$

$$= \frac{(x+5)^4 (4x-5)}{20} + C$$

$$= \int \frac{1}{u^2} \cdot \frac{du}{4}$$

$$4. \int \frac{x^3}{(1+x^4)^2} dx$$

$$u = 1 + x^4$$

$$\frac{du}{dx} = 4x^3$$

$$\frac{du}{4} = x^3 dx$$

$$= \int \frac{1}{u^2} \cdot \frac{du}{4}$$

$$= \frac{1}{4} \int u^{-2} du$$

$$= \frac{1}{4} (-1) u^{-1} + C$$

$$= -\frac{1}{4} \cdot \frac{1}{(1+x^4)} + C$$

$$= \frac{-1}{4(1+x^4)} + C$$

$$5. \int \sin^4 2x \cos 2x dx$$

$$u = \sin 2x$$

$$\frac{du}{dx} = (\cos 2x) \cdot 2$$

$$\frac{du}{2} = \cos 2x dx$$

$$= \int (\sin 2x)^4 (\cos 2x) dx$$

$$= \int u^4 \frac{du}{2}$$

$$= \frac{1}{2} \cdot \frac{1}{5} u^5 + C$$

$$= \frac{1}{10} \sin^5 2x + C$$

$$6. \int x\sqrt{4-x} dx$$

$$u = 4 - x$$

$$\frac{du}{dx} = -1$$

$$\frac{du}{-1} = dx$$

$$u = 4 - x$$

$$u - 4 = -x$$

$$-u + 4 = x$$

$$= \int (-u + 4) u^{1/2} \frac{du}{-1}$$

$$= - \int -u^{3/2} + 4u^{1/2} du$$

$$= - \left[-\frac{2}{5} u^{5/2} + 4 \left(\frac{2}{3} \right) u^{3/2} \right] + C$$

$$= \frac{2(3)u^{5/2}}{15} - \frac{4(2)(5)u^{3/2}}{15} + C$$

$$= \frac{2}{15} u^{3/2} (3u - 20) + C$$

$$= \frac{2(4-x)^{3/2} (3(4-x) - 20)}{15} + C$$

$$= \frac{2(4-x)^{3/2} (-3x - 8)}{15} + C$$