Systems of Equations

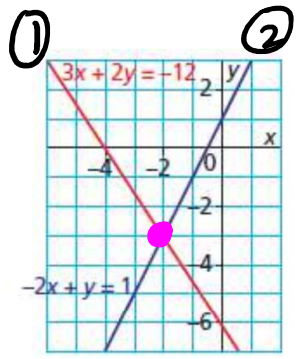
Monday, September 27, 2021 11:30 AM

Solving a System of Linear Equations Graphically

The solution of a linear system can be graphed on the same grid.
If the two lines intersect, the coordinates of the intersection are the Solution of the linear system.

For example, the following are graphed.

$3x + 2y = -12$ (1)
$-2x + y = 1$ (2)



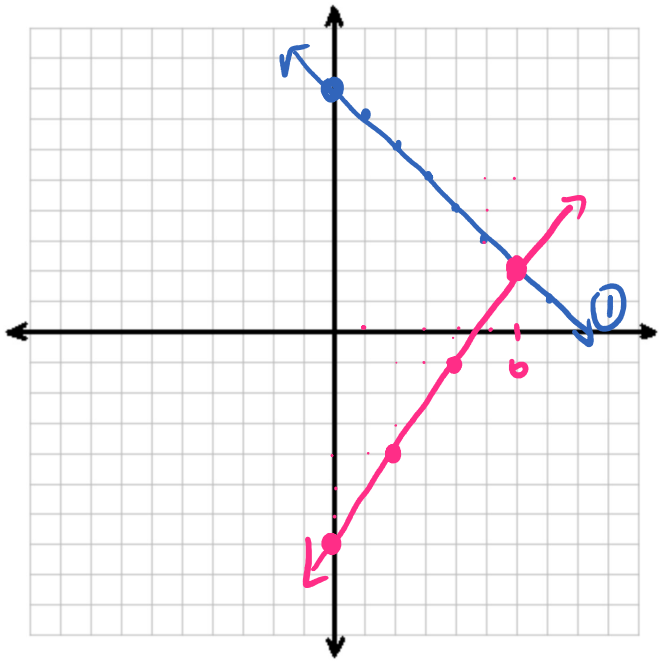
The set of points that satisfy equation (1) lie on its graph.
The set of points that satisfy equation (2) lie on its graph.
The set of points that satisfy both equations lie intersection point

The point of intersection of the graphs appears to be the point (−2, −3)

$y = mx + b$ m = slope b = y-intercept

Ex. #1: Solve this linear system by graphing.

① $x + y = 8$
② $3x - 2y = 14$



① $x + y = 8$
$y = -x + 8$
$m = -1$ ↓ →

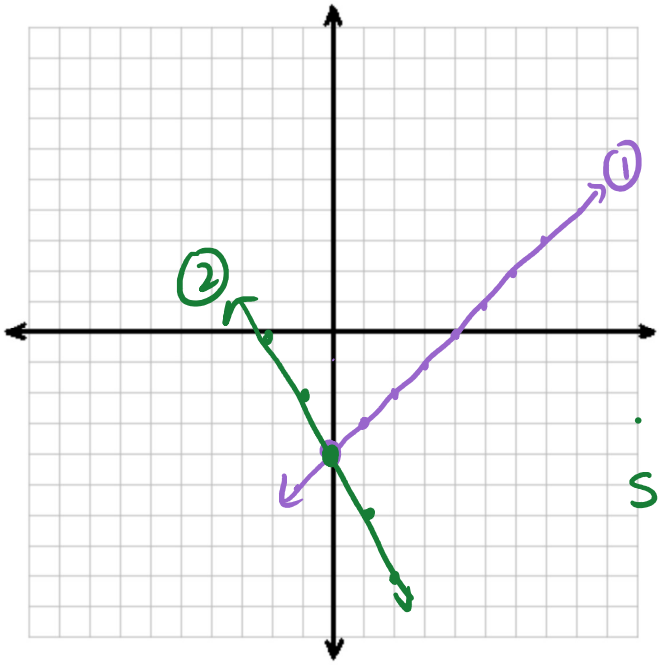
② $3x - 2y = 14$
$-2y = -3x + 14$
$y = \frac{3}{2}x - 7$
$m = \frac{3}{2}$ ↑3 →2

Solution (6, 2)

Ex. #2: Solve this linear system by graphing.

① $x - y = 4$

② $2x + y = -4$



① $x - y = 4$

$-y = -x + 4$

$y = x - 4$

$m = \frac{1}{1}$ ↑1 →1

② $2x + y = -4$

$y = -2x - 4$

$m = \frac{-2}{1}$ ↓2 →1

solution $(0, -4)$

Practice:
Solve the linear systems by graphing

1. $3x + 2y = 12$
   $x - y = -1$

2. $x + 2y = -1$
   $2x + y = -5$

1. $(2, 3)$ 2. $(-3, 1)$

Solve a System of Linear Equations by Substitution

Solving by substitution is an alternative method to solving by graphing.

Ex. #1: Solve this linear system (by substitution)

① $2x - 4y = 7$
② $4x + y = 5$

Rewrite one equation as x= or y=

② $y = -4x + 5$

Sub my equation ② into equation ①

① $2x - 4y = 7$

① $2x - 4(-4x + 5) = 7$

$2x + 16x - 20 = 7$

$18x - 20 = 7$

$18x = 27$

$x = \frac{27}{18}$

$x = \frac{3}{2}$

solved for the variable "x"

$y = -4x + 5$

$y = -4\left(\frac{3}{2}\right) + 5$

$y = \frac{-12}{2} + 5$

$y = -6 + 5$

$y = -1$

Sub our solution into an equation to find the other variable

$\left(\frac{3}{2}, -1\right)$

Ex. #2: Solve the linear system

① $x + 7y = 4$
② $2x - 9y = -15$

① $x + 7y = 4$
① $x = -7y + 4$

② $2(-7y+4) - 9y = -15$
$-14y + 8 - 9y = -15$
$-23y + 8 = -15$
$-23y = -23$
$y = 1$

$x = -7(1) + 4$
$x = -7 + 4$
$x = -3$

$(-3, 1)$

Practice:
Solve the system of equations by substitution.

3. $4x + y = -5$
   $2x + 3y = 5$

4. $x + 2y = 13$
   $2x - 3y = -9$

3. $(-2, 3)$ 4. $(3, 5)$

Solving a System of Equations by Elimination

Adding or subtracting two equations in a linear system produces equivalent linear systems. We use this property to solve a linear system by first eliminating one variable

Ex. #1: Solve this linear system by elimination.

① $2x+4y=-4$

② $-4x-5y=5$

mult by (2) →

① $4x+8y=-8$

② $-4x-5y=5$

$3y=-3$

$y=-1$

Need a set of variable with the same coefficient but different signs

Add the equations together (Eliminated one variable)

Solve for y

$2x+4y=-4$

$2x+4(-1)=-4$

$2x-4=-4$

$2x=0$

$x=0$

solution

$(0,-1)$

Ex. #2: Solve this linear system by elimination.

① $3x-4y=7$
② $5x-6y=8$

(3) ① $9x-12y=21$
(-2) ② $-10x+12y=-16$

$-1x=5$
$x=-5$

① $3x-4y=7$
$3(-5)-4y=7$
$-15-4y=7$
$+15 \quad +15$
$-4y=22$
$y=\frac{22}{-4}$
$y=-\frac{11}{2}$

$(-5,-\frac{11}{2})$

Practice:
Solve the system of equations by elimination

5. $2x+7y=24$
$3x-2y=-4$

6. $\frac{3}{4}x-y=2$
$\frac{1}{8}x+\frac{1}{4}y=2$

clear fractions
Multiply each equation by the common denominator

5. $(\frac{4}{5},\frac{16}{5})$ 6. $(8,4)$