

Unit 3 Review

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Unit 3 – Final Exam Review Exponents and Powers

Powers with rational exponents :

$$X^{\frac{m}{n}} = \sqrt[n]{X^m} \text{ or } \left(\sqrt[n]{X}\right)^m$$

Example: Write in radical form.

a) $35^{\frac{1}{2}}$

or $\sqrt[2]{35^1}$
 $\sqrt{35}$

b) $90^{0.2} = \frac{0.2 \times 10^1}{1 \times 10^1} = \frac{2^1}{10^5}$ c) $18^{\frac{6}{5}}$

$$90^{\frac{1}{5}} = \sqrt[5]{90^1} = \sqrt[5]{90}$$

$$\sqrt[5]{18^6} \text{ or } \left(\sqrt[5]{18}\right)^6$$

d) $\left(\frac{4}{9}\right)^{\frac{2}{3}}$

$$\left(\sqrt[3]{\frac{4}{9}}\right)^2$$
$$\sqrt[3]{\left(\frac{4}{9}\right)^2}$$

Example: Write each radical as an exponent.

a) $\sqrt[3]{21^1}$

$$21^{\frac{1}{3}}$$

b) $\sqrt[3]{44}$

$$44^{\frac{1}{3}}$$

c) $(\sqrt[3]{80})^2$

$$80^{\frac{2}{3}}$$

d) $\sqrt[3]{\left(\frac{4}{9}\right)^5}$

$$\left(\frac{4}{9}\right)^{\frac{5}{3}}$$

Powers that have a negative exponent

$$X^{-n} = \frac{1}{X^n}$$

Rational numbers with negative powers

$$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$$

Example: Evaluate each power with positive exponents (without a calculator)

$$\begin{aligned} \text{a) } 4^{-2} &= \frac{1}{4^2} \\ &= \frac{1}{16} \end{aligned}$$

$$\begin{aligned} \text{b) } 8^{-\frac{2}{3}} &= \frac{1}{8^{\frac{2}{3}}} \\ &= \frac{1}{(\sqrt[3]{8})^2} \\ &= \frac{1}{2^2} = \frac{1}{4} \end{aligned}$$

$$\begin{aligned} \text{c) } \left(\frac{4}{5}\right)^{-3} &= \left(\frac{5}{4}\right)^3 \\ &= \frac{5^3}{4^3} \\ &= \frac{125}{64} \end{aligned}$$

$$\begin{aligned} \text{d) } \left(\frac{4}{9}\right)^{\frac{5}{2}} &= \left(\sqrt{\frac{4}{9}}\right)^5 \\ &= \left(\frac{2}{3}\right)^5 \\ &= \frac{32}{243} \end{aligned}$$

Exponent Laws

Product of Powers	$(x^a)(x^b) = x^{a+b}$	Power of a Quotient	$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$
Quotient of Powers	$\frac{x^a}{x^b} = x^{a-b}$	Power of Zero	$x^0 = 1$
Power of a Power	$(x^a)^b = x^{ab}$	Negative Exponents	$x^{-a} = \frac{1}{x^a}$
Power of a product	$(xy)^a = x^a y^a$		

Example: Simplify and writes each expression as a single power. Write each power with positive exponents.

$$\begin{aligned} \text{a) } (2x^{-2})^5 &= 2^5 (x^{-2})^5 \\ &= 32x^{-10} \\ &= \frac{32}{x^{10}} \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{x^{-3}y^5}{xy^4} &= \frac{x^{-3}}{x^1} \cdot \frac{y^5}{y^4} \\ &= x^{-4}y^1 \\ &= \frac{y}{x^4} \end{aligned}$$

$$\begin{aligned} -3-1 \\ 5-4 \end{aligned}$$

$$\begin{aligned} \text{c) } \left(\frac{-6c^3}{\frac{1}{a^2}}\right)^{-2} &= \frac{(-6)^{-2} (c^3)^{-2}}{\left(\frac{1}{a^2}\right)^{-2}} \\ &= \frac{(-6)^{-2} c^{-6}}{a^{-1}} \\ &= \frac{a}{(-6)^2 c^6} \\ &= \frac{a}{36c^6} \end{aligned}$$

$$\begin{aligned} \text{d) } \frac{(2r^{\frac{3}{2}}s^2)^{\frac{1}{2}}(r^{\frac{1}{2}}s^{-1})}{6r^{-3}s^{-4}} &= \frac{2r^{\frac{3}{2} \cdot \frac{1}{2}} \cdot r^{\frac{1}{2}} \cdot s^2 \cdot s^{-1}}{6r^{-3}s^{-4}} \\ &= \frac{2r^2 s^1}{6r^{-3}s^{-4}} \\ &= \frac{r^2 s^1}{3r^{-3}s^{-4}} \\ &= \frac{r^{-1} s^5}{3} \\ &= \frac{s^5}{3r^1} \text{ or } \frac{s^5}{3r} \end{aligned}$$

$$\frac{3}{2} + \frac{1}{2} = \frac{4}{2} = 2$$

$$2 + (-1) = 1$$

$$2 - 3 = -1$$

$$1 - (-4) = 5$$