

PROBLEMS

In Problems 1-8 find the volume generated when the region bounded by the given curves and lines is rotated about the x-axis. (Note:  $x = 0$  is the y-axis and  $y = 0$  is the x-axis.)

1.  $x + y = 2, x = 0, y = 0$
2.  $x = 2y - y^2, x = 0$
3.  $y = 3x - x^2, y = x$
4.  $y = x, y = 1, x = 0$
5.  $y = x^2, y = 4$
6.  $y = 3 + x^2, y = 4$
7.  $y = x^2 + 1, y = x + 3$
8.  $y = 4 - x^2, y = 2 - x$

In Problems 9-22, find the volume generated by revolving the given region about the given axis.

9. The region bounded by  $y = x^4, x = 1,$  and  $y = 0$  about the y-axis.

10. The region bounded by  $y = x^3, x = 2,$  and  $y = 0,$  about the y-axis.
11. The triangle with vertices  $(1, 1), (1, 2), (2, 2),$  (a) about the x-axis; (b) about the y-axis.
12. The region in the first quadrant bounded by the curve  $x = y - y^3$  and the line  $x = 0,$  about the x-axis.
13. The region in the first quadrant bounded by  $x = y - y^3, x = 1,$  and  $y = 1,$  about the x-axis.
14. The triangular region bounded by  $2y = x + 4, y = x,$  and  $x = 0,$  about the y-axis.
15. The region in the first quadrant bounded by  $y = x^3$  and  $y = 4x,$  about the x-axis.
16. The region bounded by  $y = \sqrt{x}$  and  $y = x^2/8,$  about the x-axis.

17. The region bounded by  $y = 2x - x^2$  and  $y = x,$  about the y-axis.
18. The region bounded by  $y = \sqrt{x}, y = 2, x = 0,$  (a) about the x-axis; (b) about the line  $x = 4.$
19. The region bounded by the y-axis, and by the curves  $y = \cos x$  and  $y = \sin x$  for  $0 \leq x \leq \pi/4,$  about the x-axis.
20. The region bounded by  $y = 0$  and the curve  $y = 8x^2 - 8x^3, 0 \leq x \leq 1,$  about the y-axis.
21. The region between the curves  $y = 2x^2$  and  $y = x^4 - 2x^2$  about the y-axis.
22. The region in the first quadrant bounded by  $y = x^2, x + y = 2,$  and the x-axis, about the x-axis.
23. Use cylindrical shells and the formula  $\int x \sin x \, dx = \sin x - x \cos x + C$  to find the volume generated by

revolving about the y-axis the area bounded by  $y = 0$  and the curve  $y = \sin x, 0 \leq x \leq \pi.$

24. The area bounded by the curve  $y = x^2$  and the line  $y = 4$  generates various solids of revolution when rotated as follows:
  - a) about the y-axis,
  - b) about the line  $y = 4,$
  - c) about the x-axis,
  - d) about the line  $y = -1,$
  - e) about the line  $x = 2.$
 Find the volume generated in each case.
25. The circle  $x^2 + y^2 = a^2$  is rotated about the line  $x = b$  ( $b > a$ ) to generate a torus. Find the volume generated. (Hint:  $\int_a^b \sqrt{a^2 - x^2} \, dx = \pi a^2/2,$  since it is the area of a semicircle of radius  $a.$ )

EVEN ANSWERS:  $2.8\pi, 4.2\pi, 6.48\pi, 10.64\pi, 12.47\pi$

$14.32\pi, 16.24\pi, 18.8\pi$