Word Problems Ch 8

Thursday, November 29, 2018 9:59 AM

Solving Word Problems Using Logs

1. The number of students in a school t years after the school opens can be modelled by the equation $S = S_0[\log_2(t+1)+1]$, where S_0 is the original number of students in the school. How many years will it take for the number of students to reach 800 is the original number of students in the school was 200?

S = final # student

S_0 = original # students

t = time

$S = S_0[\log_2(t+1)+1]$

$S = 800$

$S_0 = 200$

$t = ?$

$800 = 200[\log_2(t+1)+1]$

$4 = \log_2(t+1)+1$

$3 = \log_2(t+1)$

$2^3 = t+1$

$8 = t+1$

$7 = t$

2. The temperature of hot chocolate is recorded as it cools down. Newton's law of cooling $T = 79(0.85^t)+20$ shows the temperature T degrees Celsius as a function of time t minutes. How long will it take for the hot chocolate to cool down to 40°?

T = temperature

t = time

$T = 79(0.85^t)+20$

$T = 40^\circ$

$t = ?$

$40^\circ = 79(0.85^t)+20$

$20 = 79(0.85)^t$

$\frac{20}{79} = 0.85^t$

$\log\frac{20}{79} = \log 0.85^t$

$\log\frac{20}{79} = t\log 0.85$

$t = \frac{\log\frac{20}{79}}{\log(.85)}$

$t = 8.45$ min

3. Sound is measured in decibels (dB). The level of sound, L, is given by $L = 10\log\frac{I}{I_0}$, where I is the intensity W/m^2 of the sound and I_0 is the faintest sound detectable to humans. A sound engineer increases the volume at a concert from 90 decibels (dB) to 93 (dB). Show that this increase approximately doubles the intensity of the sound.

L = level of sound
I = Intensity
I_0 = given #

$$L = 10\log\frac{I}{I_0}$$

Loud
$L = 93$
$93 = 10\log\frac{I}{I_0}$
$9.3 = \log\frac{I}{I_0}$
$10^{9.3} = \frac{I}{I_0}$

Quiet
$L = 90$
$90 = 10\log\frac{I}{I_0}$
$9 = \log\frac{I}{I_0}$
$10^9 = \frac{I}{I_0}$

$$\frac{\text{Loud}}{\text{Quiet}}$$
$$\frac{\frac{I_L}{I_0}}{\frac{I_Q}{I_0}} = \frac{10^{9.3}}{10^9}$$
$$\frac{I_L}{I_Q} = \frac{10^{9.3}}{10^9} = 10^{.3}$$
$$= 1.99$$
$$= 2$$

4. A major earthquake of 7.5 is 375 times as intense as a minor earthquake. Find the magnitude (Richter scale) of the minor earthquake.

$M = \log\frac{A}{A_0}$ M = magnitude
A = Amplitude ground motion

Major
$M = 7.5$
$7.5 = \log\frac{A}{A_0}$
$10^{7.5} = \frac{A}{A_0}$

Minor
$M = X$
$X = \log\frac{A}{A_0}$
$10^X = \frac{A}{A_0}$

$$\frac{\text{Amp Major}}{\text{Amp Minor}} = 375$$
$$\frac{A_{maj}}{A_{min}} = \frac{10^{7.5}}{10^X} = 375$$
$$10^{7.5-X} = 375$$

$\log_{10} 375 = 7.5 - X$
$X = 7.5 - \log 375$
$X = 4.9$