Pre-Calculus 12 Name: _____

Pre-Calculus 12: Final Review

Chapter 1 &2 Transformations

1. In what order should transformations be applied to a graph?

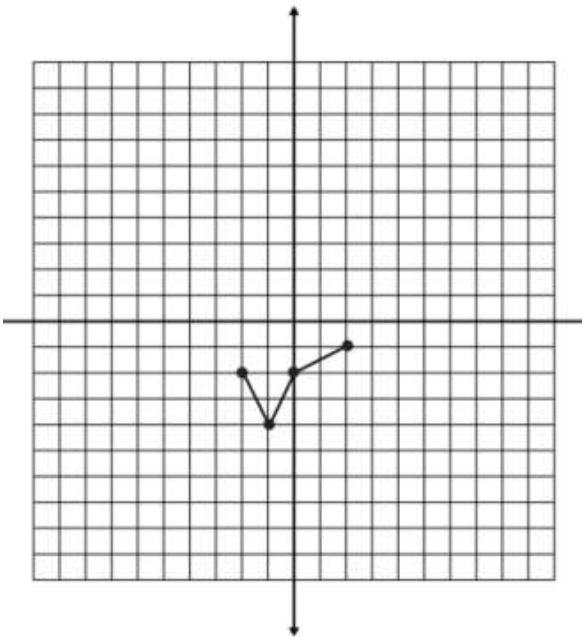
2. Describe the transformations in each equation in an appropriate order.

a) $2y - 8 = 6f(x - 2)$

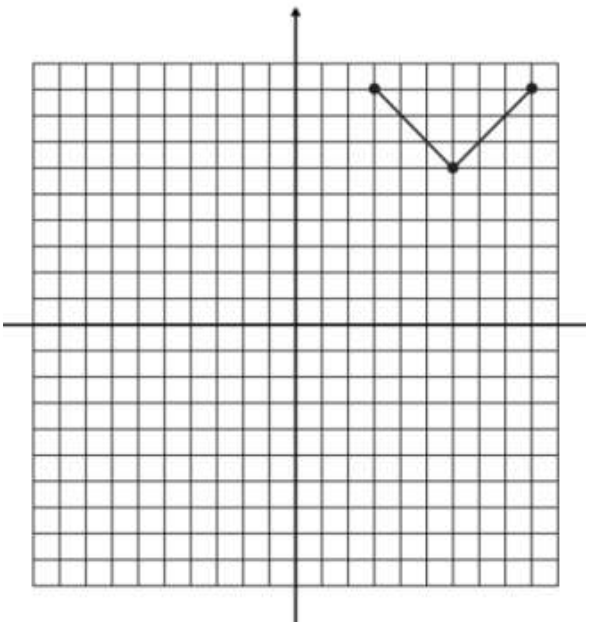
b) $y = -3f[-4(x - 1)] + 2$

3. Draw the transformations of each graph.

a) $y = f(-\frac{1}{4}x) + 1$



b) $f(x) = 2f(3x - 6) - 10$



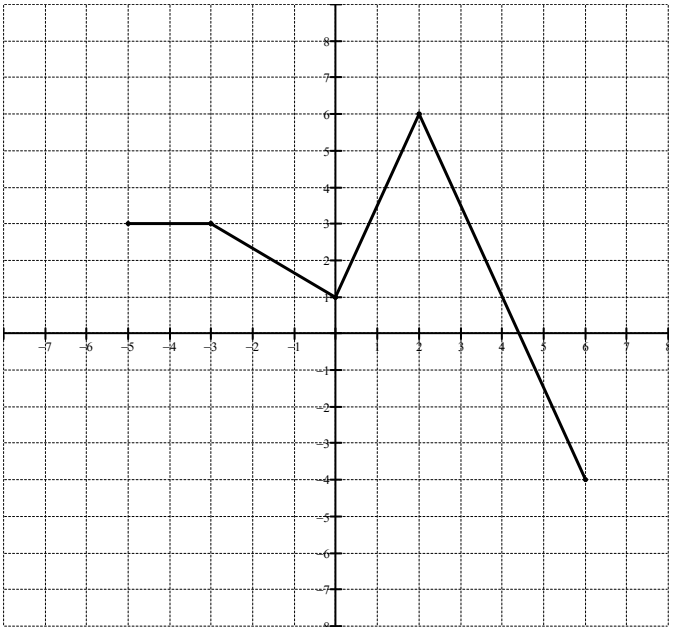
4. The following transformations are applied to a function $y = f(x)$

Vertical stretch by a factor of 4
Horizontal stretch by a factor of 3
Reflection over the x-axis
Translated 2 up, 5 to the left

a) Create a mapping notation for the transformations

b) If the point (-2,5) is on f(x), use the mapping notation to find the new point after the transformations are applied.

5. Sketch the inverse of the relation.

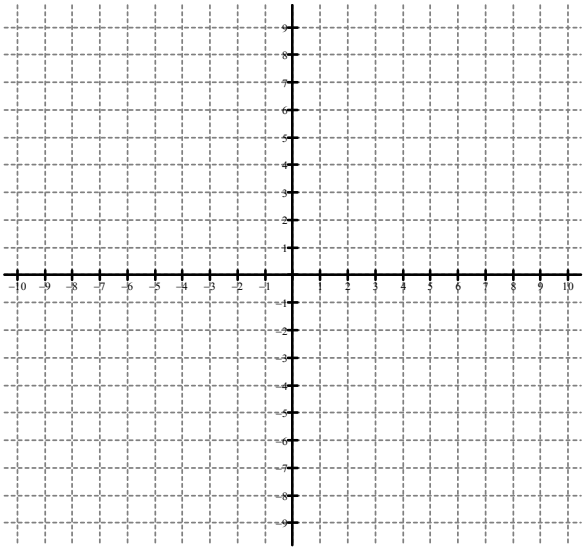


6. Find the inverse of $f(x) = \frac{3}{x-2}$

7. The domain and range of a function are: $\{x|-3 \leq x \leq 6 \quad x \in R\}$ $\{y|y > 7 \ \ y \in R\}$ State the domain and range of the inverse.

8. Sketch the graph of the function using transformations. List the transformations in an appropriate order. State its domain and range.

$y = 2\sqrt{x-3} + 4$



Domain:

Range:

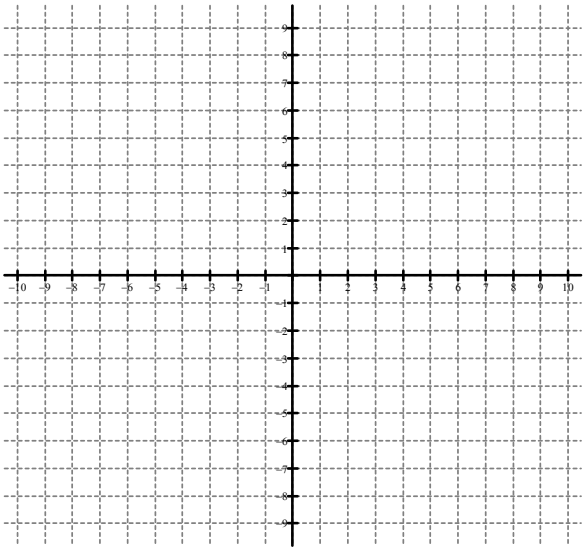
9. Write a **single equation** for a radical function with the given domain and range.

D: $\{x / x \geq 3, x \in \Re\}$

R: $\{y / y \leq -5, y \in \Re\}$

10. Solve the equation graphically.

$2\sqrt{x+2} = 1 - x$



Chapter 3 Polynomials

1. For the following polynomial function, state the following:

$f(x) = x^4 - 5x^3 + 2x^2 + 20x - 24$

a) degree _____

b) type _____

c) leading coefficient _____

d) constant term _____

e) the value of the y-intercept _____

f) maximum possible number of x-intercepts _____

g) end behavior of the corresponding graph_____

2. Use the Factor Theorem to determine whether $x^4 - 2x^3 + 3x - 4$ has a factor of $x - 2$

3. For the following function determine a) the x-intercepts, b) the degree c) end behavior of the graph, d) the zeroes and their multiplicity, e) the y-intercept of the graph, and f) the intervals where the function is positive and g) the intervals where the function is negative.

$$f(x) = x^4 + 4x^3 - 7x^2 - 34x - 24$$

y-intercept *degree and end behavior*

x-intercepts *zeroes and multiplicity*

intervals of positive and negative

4. Find the value of "k" if the remainder is 3 when $x^3 - x^2 + kx - 15$ is divided by $x - 2$.

Chapter 4 Trigonometry and the Unit Circle

1. Change the given angle from radians to degrees or vice-versa.

a) $\frac{5\pi}{9}$ b) $240°$

2. Find one positive and one negative co-terminal angle for the original angles in question #1.

3. A circle as central angle of $40°$ and a radius of 7ft. Find the arclength of the sector.

4. A radius of a circle is 8 cm, and the length of an arc on the circle is 12 cm. In radians, what is the central angle that subtends this arc length?

5. The point P(x,y) is located where the terminal arm of angle θ and the unit circle intersect. Determine the coordinates of point P if :

a) $\theta = 210°$

b) $\theta = \frac{3\pi}{4}$

6. Identify a measure for the central angle θ in the interval $0 \le \theta \le 2\pi$ such that $P(\theta)$ is the given point.

a) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

b) $\left(1, -\sqrt{3}\right)$

7. Solve algebraically for the domain stated. $0 \le x < 2\pi$. Answer using exact values.

$5 \sin \theta + 2 = 1 + 3 \sin \theta$

Chapter 5 Trigonometric Functions and Graphs

1. Determine the key features for the function $y = -5sin\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right) + 15$

a) Amplitude: _____ b) Period: _____

c) Phase Shift: _____ d) Vertical displacement: _____

e) Domain: _____ f) Range: _____

2. Write the equation of each sine function in the form $y = asinb(x - c) + d$ given its characteristics.

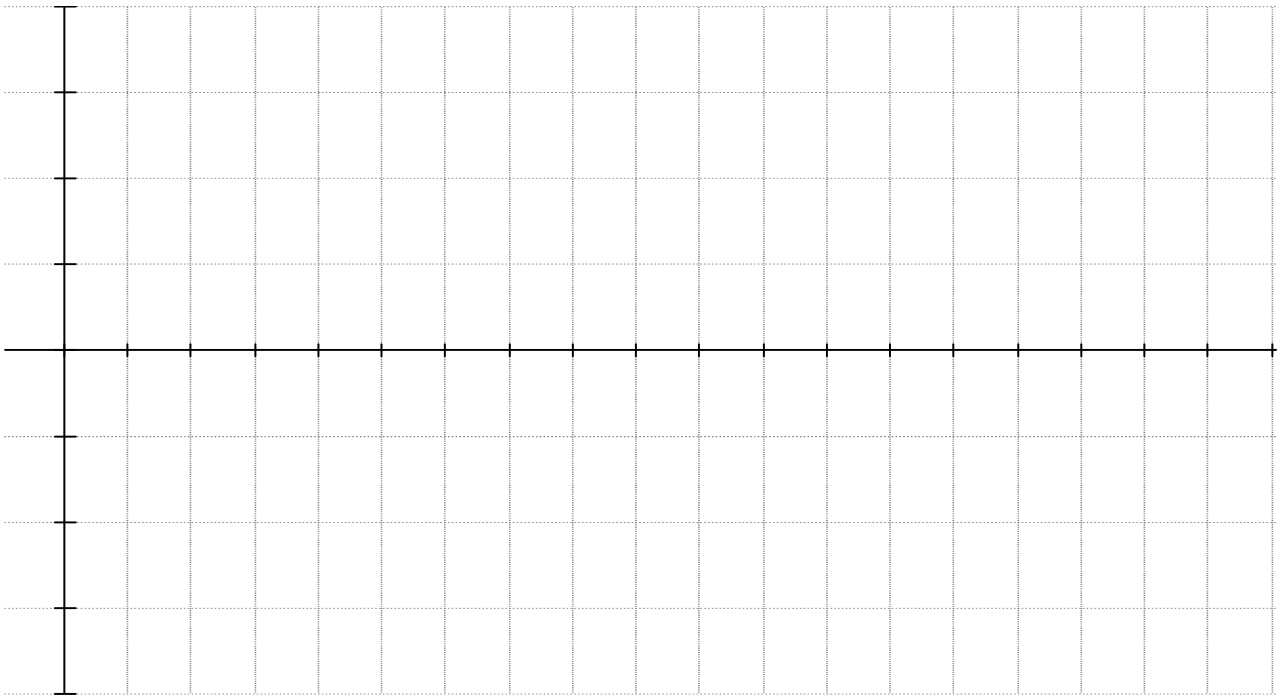
a) amplitude 2, period π, phase shift $\frac{\pi}{3}$ to the left, vertical displacement 1 unit down

b) amplitude $\frac{1}{4}$, period 6π, phase shift π to the right, vertical displacement 2 units up.

3. Graph the following function (show 2 periods) State the period and phase shift

$y = 2cos\frac{1}{2}\left(x - \frac{\pi}{2}\right) + 2$

period : _____ phase shift: _____



4. Solve the following trigonometric equations algebraically, using exact values. Show all work.

a) $4\sin(x - \frac{\pi}{3}) = -2 \qquad 0 \leq x < 2\pi$

b) $2sin^2x + 5\sin x - 3 = 0 \qquad 0 \leq x < 2\pi$

Chapter 6 Trigonometric Functions and Identities

1. Simplify the following:

a) $\cos(\alpha + 90°)$

b) $sin25°cos65° + cos25°sin65°$

2. Solve the following, accurate to 2 decimal places for $0 \leq \theta < 2\pi$.

a) $2sec^2x + 5secx - 3 = 0$

b) $2cos^2x = -3sinx$

3. Solve for all possible solutions in radians. (Find a general solution)

$sin\ 2x = 2sinx$

4. Use sum or difference identities to find the exact value of each trigonometric expression.

a) $sin15°$ b) $tan165°$

5. Simplify the following:

a) $cot^2x\ sin^2x + cos^2x$ b) $\frac{sec\theta - cos\theta}{csc\theta - sin\theta}$

c) $(1 + \cos\theta)(\csc\theta - \cot\theta)$

6. Prove the identity.

a) $\sin^3 x + \sin x \cos^2 x = \sin x$

b) $\dfrac{1+\cos x+\cos 2}{\sin x+\sin 2} = \cot x$

c) $\dfrac{\sin 2x}{2 - 2\cos^2 x} = \cot x$

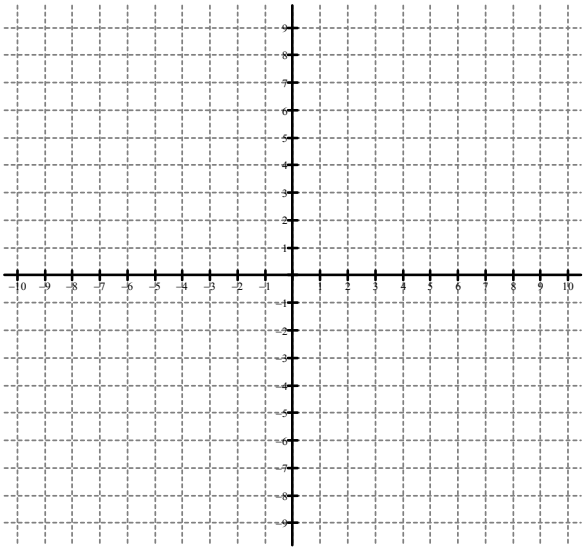
d) $\dfrac{\cot x}{\csc x - 1} = \dfrac{\csc x + 1}{\cot x}$

Chapter 7 Exponential Functions

1. Sketch the graph of each function using transformations and tables of values. List the transformations in an appropriate order. (4 marks each)

(a) $y = -2(2)^{x-1} + 4$

- _____
- _____
- _____
- _____



2. Solve

a) $64^{4x} = 16^{(x+5)}$

b) $36^{-3n} \cdot 216 = \left(\frac{1}{216}\right)^{-2n}$

c) $\frac{9^{3x}}{243^{-x-1}} = 81^{2x}$

3. The half –life of sodium-24 is 17 hours. A chemistry teacher has 40 mg of sodium-24. After how long will only 5 mg remain?

4. A bacteria culture contains 6250 bacteria and doubles every 3 hours. What was the population 9 hours ago?

5. The initial count was 530 bacteria in a culture. Ten hours later, there were 14310 bacteria. What is the tripling period for this type of bacteria?

Chapter 8 Logarithmic Functions

1. For the equation $y = 3\log_5(6(x+2)) - 4$, state:

a) domain

b) range

c) equation of the asymptote

d) x-intercept (if it exists)

e) y-intercept (if it exists)

2. Simplify to a single log and then evaluate if possible.

a) $2\log_2 12 - (\log_2 6 + \frac{1}{3}\log_2 27)$

b) $2\log_5 4 + \log_5 3 - \log_5 11$

c) $\log x - 3\log y + \frac{2}{3}\log z$

d) $\log_2(x+2) + \log_4 x$

3. Solve, answer to nearest hundredth if necessary.

a) $\log_7(2x-3) - \log_7(x+2) = 1$

b) $\log_b(x+2) - \log_b 4 = \log_b 3x$

c) $2\log_4(x+4) - \log_4(x+12) = 1$

d) $2\ln(5x-2) = 16$

4. Solve, answer to nearest hundredth if necessary.

a) $9^{2x-1} = 71^{x+2}$

b) $4(7^{x+2}) = 9^{2x-3}$

c) $e^{3x+1} = 2$

Chapter 9 Rational Functions

1. For each function, find the locations of any vertical asymptotes, points of discontinuity, and intercepts.

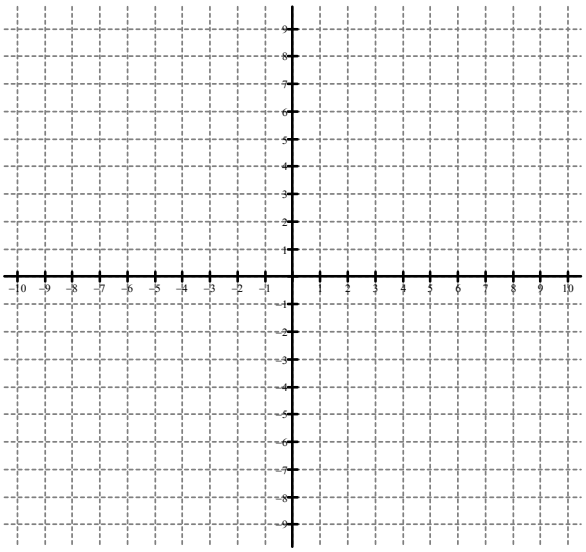
a) $y = \frac{x^2 + 4x}{x^2 + 9x + 20}$

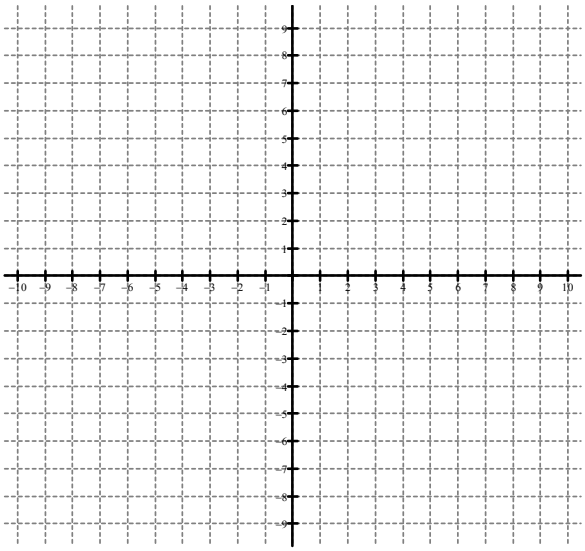
b) $y = \frac{2x^2 - 5x - 3}{x^2 - 1}$

2. Graph the functions

a) $y = \frac{-2}{x+3} + 1$

b) $y = \frac{4x-5}{x-2}$





Chapter 10 Composite Functions

1. If $f(x) = \sqrt{x+2}$ and $g(x) = |2x|$, find $f \circ g(-7)$

2. $f(x) = x^2 + 7$ and $g(x) = 2x - 1$ find $f\big(g(x)\big)$

Chapter 12 Series

1. How many terms are in the sequence $2, 6, 18, \ldots, 486$

2. The sum of an infinite geometric series is 63 and the first term is 21. Find the common ratio.

3. Find the sum of the first 12 terms of the series. $12 + 4 + \frac{4}{3} \ldots$